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An approach to incomplete multiplicative preference relations and its application in group decision making

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ABSTRACT

To measure the consistency of multiplicative preference relations, a reasonable concept of consistent multiplicative preference relations is defined, and the multiplicative geometric consistent index (MGCI) is presented. In a simulation method, the average values of the MGCI are studied. Considering incomplete multiplicative preference relations, a consistency-based linear programming model is constructed, which is optimal from the point of view of consistency with respect to the known judgments. In a group decision making context, the group consensus index (GCOI) is given to measure the consensus of individual multiplicative preference relations. Then, the hybrid weighted geometric mean (HWGM) operator is defined to calculate the collective multiplicative preference relation. Based on the consistency and consensus analysis, an approach to group decision making with incomplete multiplicative preference relations is developed. Furthermore, we briefly research the application of the new method to incomplete interval multiplicative preference relations and incomplete fuzzy preference relations. Meanwhile, the associated examples are offered to show the practicability and efficiency of the proposed procedure.

1. Introduction

Because the analytic hierarchy process (AHP) [57] can clearly address complex decision making problems, it has been commonly used in many fields [1,25,42,43,51,59,77,85]. In the AHP, multiplicative preference relations are often used by decision makers (DMs). Similar to the other preference relations, the main issue is how to obtain the priority vector. As we know, the consistency of preference relations influences the final ranking order of alternatives. Saaty [66] first noticed this issue and gave a consistency ratio (CR) to judge the consistency of multiplicative preference relations, which considers a multiplicative preference relation to be acceptably consistent if $CR < 0.1$. Lance and Verdini [49] investigated the distribution of random inconsistency and decision rule implications. The authors suggested that there should be stricter consistency requirements for three and four order multiplicative preference matrices. Crawford and Williams [13] proposed the geometric consistency index (GCI) based on the row geometric mean method (RGMM). Recently, Dong et al. [19] introduced two consistency indices for multiplicative preference relations: one consistency index uses the logarithmic Manhattan distance; the other consistency index is based on the logarithmic Euclidean distance metric. Then, Dong et al. [19] defined the consistency index of interval multiplicative preference relations, and constructed a linear programming model for improving

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the consistency of interval multiplicative preference relations. Furthermore, Xu and Wei [79] developed a method to improve the consistency of multiplicative preference relations. More studies regarding improving the consistency can be observed in the literature [12,26,28,52,62,76,78].

To obtain a complete preference relation, a DM needs to compare each pair of objects and provide their preference values. However, in some practical decision making problems, the DM may only offer partial preference values for various reasons such as time pressure, lack of knowledge and information, and the limited expertise related to the problem domain. This is the so-called incomplete preference relations [48]. Xu [80] defined several concepts for incomplete preference relations such as incomplete reciprocal relations, additive consistent incomplete reciprocal relations and multiplicative consistent incomplete reciprocal relations. Then, the author built two goal programming models to derive the priority vector from incomplete reciprocal relations. Based on the concept of additive consistency [70], Herrera-Viedma et al. [33] proposed an iterative procedure to estimate the missing values of incomplete fuzzy preference relations, which only uses the available values provided by the same DM. Because this method can only cope with the situation where each alternative is compared at least once, Lee [50] presented another method to estimate the missing values of incomplete fuzzy preference relations. This method considers the unreachable missing values to be zero. Later, Chen et al. [11] noticed that the missing values obtained by Lee's method may not fall within [0, 1] and presented another approach to estimate the missing values. In this method, the authors endowed the unreachable missing values with 0.5. However, the same issue as in Lee's method also exists in Chen et al. method. To cope with the situation where ignorance alternatives exist, namely, all preference values involving one alternative are unknown, Alosnso et al. [3] introduced two methods (individual strategies and social strategies) to estimate the missing values, which extend the application of the Herrera-Viedma et al. approach [33]. In the first method, the authors estimated the missing values by assuming the preference degree of the unknown alternative over another being indifference, namely, 0.5, or the unknown alternative being similar to another. In the second method, it uses the preference values from the other DMs. However, both of these methods need additional information. Similar to Herrera-Viedma et al. [33], Chen et al. [7] presented another approach to estimate the missing judgments, but the missing values may also fail to belong to $[0,1]$. To address the situation where the DM has no information on alternatives, Meng and Chen [57] built a consistency-based goal programming model to estimate the missing values, which can guarantee the missing values belong to $[0,1]$. Based on the consistency index [33] and the consensus consistency index [14], Zhang et al. [86] developed a method to group decision making with fuzzy preference relations by using two linear programming models. Furthermore, the authors introduced a linear programming model to calculate the missing values. Gong [31] developed a least-square method to derive the priority vector of group decision making with incomplete fuzzy preference relations. Fedrizzi and Giove [30] proposed a method for calculating the missing values of incomplete fuzzy preference relations by minimizing the global inconsistency. Later, Chiclana et al. [15] analyzed the methods in [30,33] and noted that these two methods are complementary rather than competitors. Chiclana et al. [16] applied a functional equation to model the cardinal consistency of fuzzy preference relations. The authors noted that cardinal consistency with a conjunctive representable cross ratio uninorm is equivalent to Tanino's multiplicative transitivity property. Furthermore, Chiclana et al. [16] developed a method to construct perfect cardinal consistent fuzzy preference relations by using the cross ratio uninorm based on $n-1$ preference values. Similar to [33], Alonso et al. [4] defined additive consistent fuzzy linguistic preference relations by using the 2-tuple linguistic model and introduced a method to estimate the missing linguistic values. After that, the authors developed a group decision making method with incomplete fuzzy linguistic preference relations. After the pioneer work of Alonso et al. [4], Cabrerizo et al. [9] developed a method for group decision making method under an incomplete unbalanced fuzzy linguistic environment, while Alonso et al. [5] studied Web 2.0 communities by using linguistic preference relations and developed a consensus-based group decision making method. Based on Tanino's multiplicative transitivity property [71], Alonso et al. [6] developed a web-based group decision making method with different formats of preference relations. This method is based on both consistency and consensus measures. Meanwhile, the authors introduced a consensus/consistency control mechanism by which the feedback mechanism is offered to change undesirable preference values and estimate the missing values. Furthermore, Pérez et al. [63] developed a mobile-based dynamic group decision making method. This method has two main characteristics: (1) the DMs can express their judgments by using different preference relations; and (2) the number of alternatives may be changed in the process of decision making.

In addition to fuzzy preference relations [60] and linguistic preference relations [7,20,81], multiplicative preference relations [65] are another important type of preference relation. Although researchers argued that there are many reasons to use fuzzy preference relations and Kacprzyk and Fedrizzi [44] showed that any multiplicative preference relation can be converted into a fuzzy preference relation, there are several characteristics of multiplicative preference relations. Furthermore, any fuzzy preference relation can also be transformed into a multiplicative preference relation. This means that the ranking order of objects with a fuzzy preference relation can be derived from researching the associated multiplicative preference relation. As Herrera-Viedma et al. [35] noted, the additive transitivity for fuzzy preference relations [70] corresponds to the multiplicative transitivity for multiplicative preference relations [66]. Furthermore, the additive transitivity for linguistic preference relations based on the 2-tuple linguistic model [7] coincides with the additive transitivity for fuzzy preference relations [70]. It is worth noting that Alonso et al. [7] introduced an approach to estimate missing values for these three types of preference relations. As one can see in [33], this method can only cope with incomplete preference relations where each alternative is compared at least once.

The purpose of this paper is to develop a method to cope with the situation where ignorance alternatives exist in multiplicative preference relations. To do this, we first introduce a consistency measure for multiplicative preference