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# Transcale control for a class of discrete stochastic systems based on wavelet packet decomposition

Lin Zhao *, Yingmin Jia

*The Seventh Research Division, Beihang University (BUAA), Beijing 100191, PR China*

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ABSTRACT

In this paper, a wavelet packet decomposition (WPD) based real-time transcale linear-quadratic-Gaussian (LQG) tracking control algorithm is given for a class of discrete stochastic systems, which is developed with the state-space models of wavelet packet coefficients at the coarsest scale decomposition layer established by Haar WPD. The WPD-based transcale control algorithm provides a compromise between performance index and computational efficiency compared with the conventional LQG tracking control algorithm, and it is an improvement of the wavelet transform based algorithm previously proposed. Simulation results are presented to demonstrate the effectiveness of the proposed algorithm.


## 1. Introduction

The linear-quadratic-Gaussian (LQG) control is an optimal control of linear systems with quadratic cost function in the presence of Gaussian noises, which has been deeply researched and many results have been obtained [1,2,16,21,22,37,20]. For example, You et al. [37] discussed the quantised LQG control for linear stochastic systems; Arantes et al. [20] investigated the application of LQG control in spacecraft attitude systems. Output tracking problem is one of the most common and important issues in designing a control system, which has wide applications in dynamic processes in industry, economics, and biology [15,8,31,38,4,39,40,36,41,9,27,33,10]. The main objective of tracking control is to make the output of the model, via a controller, track the output of a given reference model as closely as possible. The LQG-based tracking control algorithms in [8,31,38,4] can be regarded as the classical single scale system control method since they focus on system represented at a single scale and design the control law at that scale. In practice, however, the system or output may contain multiscale (or equivalently, multiresolution) features. Even if they do not have multiscale features, more confidence can be obtained by using the multiscale processing algorithm, which can reduce the uncertainty and complexity of the problems [5]. Moreover, the speed up computation of control actions have been studied in some optimal control to guarantee the complex control schemes in real time [34,14]. In our initial work [45], a wavelet transform (WT)-based transcale LQG tracking control algorithm was proposed, which brought LQG control into a multiscale scenario and effectively improved the computation efficiency of conventional LQG algorithm by parallel processing.

Models that describe process behavior at several spatial and temporal scales are essential for many engineering tasks ranging from process analysis and design to operational monitoring, control and optimization [29]. Multiscale processing algorithm is an effective tool to represent the phenomena occurred at different scales, see [23,25,5,11,12,17,42,18,28,19,6,7,32,43,29,30,26]. The

* Corresponding author.
*E-mail addresses:* zhaolin1585@163.com (L. Zhao), ymjia@buaa.edu.cn (Y. Jia).

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wavelet transform (WT), which is a tool for describing multiscale data structures, has been used to develop multiscale processing algorithm. For example, Zhao et al. [43] used WT to develop a robust transcale state estimation algorithm for multiresolution discrete-time systems, which could not only fuse multiresolution sensor information but also improve the computation efficiency and estimation accuracy; Stephanopoulos et al. [29,30] introduced an alternative philosophy to establish the multiscale models by Haar WT and develop the multiscale model predictive control algorithm, which could also improve the computation efficiency.

Most of these approaches in [43,29,30,26,45] rely on the multiscale decomposition to systems or measurements through WT, but sometimes it is not an optimal choice for discrete sequences compared with wavelet packet decomposition (WPD) [13]. WPD is a transformation in which the signal passes through more filters than WT. For $J$ scales decomposition, the WPD produces $2^J$ sets of coefficients as opposed to $J+1$ sets of coefficients in WT. However, due to the downsampling process the overall number of coefficients is still the same without redundancy. From the point of view of signal analysis, the standard WT may not produce the best result, since it is limited to wavelet basis [35,13]. In fact, the wavelet basis based WT is only a special WPD. From the point of view of computation, the decomposition structure of WPD gives us more chances to improve the computation efficiency compared with that of WT. So it is necessary to further study the WPD-based algorithm for control and estimation of discrete stochastic systems, which can ensure the multiscale processing algorithm be better combined with conventional control algorithm.

Motivated by the above discussions, in this study, we introduce a WPD-based real-time transcale LQG tracking control algorithm for the output tracking of discrete stochastic systems. The term *transcale control* is defined in [44,45], which is applied to emphasize the fact that such a control strategy is capable of designing the control law at the finest scale but reflecting the desired performances at different coarser scales. The Haar WPD is employed to establish the state-space models of wavelet packet coefficients (WPC) at the coarsest scale decomposition layer. Based on these models, the real-time transcale LQG tracking control algorithm is proposed by three steps. The computational efficiency analysis is given. It can be proved that the control law designed using the proposed algorithm has the desired performance in the meaning of transcale control.

In summary, the contributions of this paper are as follows:

(1) The LQG tracking control algorithm is further developed in the multiscale framework. This WPD-based algorithm can improve the computation efficiency compared with the LQG tracking control algorithm.
(2) The WPD is firstly proposed into the multiscale control for discrete stochastic systems (compared with [29,30,45]).
(3) The transcale LQG tracking control algorithm in [45] is developed based on WT while the decomposition under WT is a special WPD, so the WT-based algorithm in [45] is only a special case of the WPD-based algorithm developed in this paper.
(4) From the point of the view of computation efficiency, the decomposition structure in this paper is better than that of others, so the WPD-based algorithm can effectively improve the computation efficiency compared with the WT-based algorithm in [45].

The rest of this paper is organized as follows. The control problem formulation is summarized in Section 2. A brief introduction about the Haar WPD of discrete sequences is given in Section 3. The WPD-based control scheme is presented in Section 4 numerical example of the derived control design is presented in Section 5. Conclusions are given in Section 6.

## 2. Problem formulation

Consider the following discrete stochastic system

$$x_{k+1} = Ax_k + Bu_k + Dw_k \tag{1}$$

$$y_k = Cx_k + v_k \tag{2}$$

$$z_k = C_d x_k, \quad k = 0, 1, \ldots, N \tag{3}$$

where $x_k \in \mathrm{R}^n$, $u_k \in \mathrm{R}^m$, $y_k \in \mathrm{R}^q$ and $z_k \in \mathrm{R}^s$ are the state, control input, measurement, and output vectors respectively. $A \in \mathrm{R}^{n\times n}$, $B \in \mathrm{R}^{n\times m}$, $D \in \mathrm{R}^{n\times p}$ are the system matrices, $C \in \mathrm{R}^{q\times n}$ is the observation matrix, $C_d \in \mathrm{R}^{s\times n}$ is the output matrix. The system noise $w_k \in \mathrm{R}^p$ and the measurement noise $v_k \in \mathrm{R}^q$ are uncorrelated white zero-mean Gaussian vectors with covariance matrices $UandV$ respectively. The statistic characteristics of initial $x_0$ are given as $E\{x_0\} = m_0 \in \mathrm{R}^n$, $E\{(x_0 - m_0)(x_0 - m_0)^T\} = \overline{P}_0 \in \mathrm{R}^{n\times n}$.

In this paper, the finite-horizon output tracking problem is considered, where the desired output is given by $\bar{z}_k \in \mathrm{R}^s$ , the time horizon $N = M2^J - 1$ and $M$ is a positive integer. Denote

$$e_k \triangleq \bar{z}_k - z_k \tag{4}$$

The LQG tracking control can solve the above problem, which is to find $u_k$ such that the following cost function is minimized

$$J(u_k) \triangleq \frac{1}{2}E\left\{e_N^T F e_N + \sum_{k=0}^{N-1}(e_k^T Q e_k + u_k^T R u_k)\right\} \tag{5}$$