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A note on Hamiltonian paths and cycles with prescribed edges in the 3-ary n-cube ☆

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ABSTRACT

In Wang et al. (2011), the authors proved that the 3-ary n-cube admits a Hamiltonian path between u and v passing through every edges in a prescribed set $\mathcal{P}$ with at most $2n-2$ edges if the subgraph induced by $\mathcal{P}$ consists of pairwise vertex-disjoint paths, none of which has u or v as internal vertices or both of them as end-vertices. They proved the above results based on some Lemmas including Lemma 4.7 in the paper. However, the proof of Case 2.4 of Lemma 4.7 is incorrect. In this note, we will give a counterexample and provide a correct proof. Therefore, the main result in the above paper is still correct.

1. Introduction

The k-ary n-cube Q_n^k has been an important interconnection networks for massively parallel systems. The problem of path and cycle embedding has been involved in the area of graph theory and parallel and distributed systems because some parallel applications such as those in image and signal processing are originally designated for a path or cycle architecture. Wang et al. [5] investigated the problem of embedding Hamiltonian paths and cycles passing through prescribed edges into 3-ary n-cubes. Given a 3-ary n-cube with $n \geqslant 2$, let $\mathcal{P}$ be a set of at most $2n-2$ prescribed edges in the 3-ary n-cube and let $u,\ v$ be two vertices such that the subgraph induced by $\mathcal{P}$ consists of pairwise vertex-disjoint paths, none of which have u or v as internal vertices or both of them as end-vertices. They constructed a Hamiltonian path of the 3-ary n-cube between u and v passing through $\mathcal{P}$ by proving some Lemmas including the following.

Lemma 1.1 (see Lemma 4.7 in [5]). *If $\mathcal{P} \setminus (\mathcal{P}^0 \cup \mathcal{P}^1 \cup \mathcal{P}^2) = \{e\},\ |\mathcal{P}^0| \leqslant 2n-4$ and $u,\ v$ are in different subcubes, then Q_n^3 has a Hamiltonian path $\pi[u, v]$ passing through $\mathcal{P}$.*

However, the proof of Case 2.4 of Lemma 4.7 is incorrect. What is perhaps worse, the main result in [5] has been used to obtain further results by some other researchers (see for example [2,6]). In this note, we will give a counterexample and provide a correct proof. Therefore, the main results in [5] and literatures citing [5] are still correct. The rest of this note is organized as follows. In Section 2, we introduce some definitions and results. In Section 3, we will give a counterexample to the proof of Case 2.4 of Lemma 4.7 in [5]. Revised proof will be given in Section 4.

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2. Preliminaries

In this note, terminology and notation mostly follow [1,5]. The term n always mean an integer not less than 3, unless otherwise stated.

Let $u,\ v$ be two distinct vertices and $\mathcal{P}$ be a linear set in Q_n^3. We say that $\{u, v\}$ and $\mathcal{P}$ are *compatible* (or, u and v are compatible for $\mathcal{P}$) in Q_n^3 if both u and v are incident with at most one edge of $\mathcal{P}$ and there is no path in $\langle\mathcal{P}\rangle$ between u and v, where $\langle\mathcal{P}\rangle$ is the subgraph induced by $\mathcal{P}$ in Q_n^3.

The 3-ary n-cube can be partitioned into three disjoint Q_{n-1}^3's $Q[0],\ Q[1]$ and $Q[2]$ by deleting all the edges in some dimension. For $i \in \{0,1,2\}$, let $V^i = V(Q[i])$ and $E^i = E(Q[i])$. For a vertex u^i in $Q[i]$, the corresponding vertex of u^i in $Q[j]$ is denoted by u^j. Let $\mathcal{P} \subseteq E(Q_n^3)$ be a linear set and let $\mathcal{P}^i = \mathcal{P} \cap E(Q[i])$ for $i \in \{0,1,2\}$. We say that an edge (u^i, v^i) of $Q[i]$ is *free* for $\mathcal{P}^i$ if $(u^i, v^i) \notin \mathcal{P}^i$ and $\mathcal{P}^i \cup \{(u^i, v^i)\}$ is a linear set. Furthermore, if (u^j, v^j) is also free for $\mathcal{P}^j$, we say that (u^i, v^i) is *(i,j)-free*, where $i,j \in \{0,1,2\}$.

In the remainder of this note, let $\mathcal{P} \subseteq E(Q_n^3)$ be a linear set with $|\mathcal{P}| = 2n-2$. Then there exists a partition $Q[0],\ Q[1]$ and $Q[2]$ of Q_n^3 such that $|\mathcal{P} \setminus (\mathcal{P}^0 \cup \mathcal{P}^1 \cup \mathcal{P}^2)| \leqslant 1$. Let u, v be two vertices such that $\{u, v\}$ and $\mathcal{P}$ are compatible in Q_n^3.

In [5], the main result was proved by induction on n and the base case has been dealt with successfully. Suppose, as the induction hypothesis, that for an arbitrary prescribed edge set with at most $2n-4$ edges in $Q[i]$ and any two distinct vertices which are compatible for the prescribed edge set, there is a Hamiltonian path between these two vertices passing through the prescribed edge set, where $i \in \{0,1,2\}$.

The following two lemmas will be used in the revised proof of Lemma 1.1 in Section 4.

Lemma 2.1 ([3,4]). *Let $F \subset V(Q_n^3)$ with $|F| \leqslant 2n-3$, where $n \geqslant 2$. For any two distinct vertices $u, v \in V(Q_n^3) \setminus F$, there is a Hamiltonian path between u and v in $Q_n^3 - F$.*

Lemma 2.2 (See Lemma 3.6 in [5]). *Let $x^i \in V^i$ and $y^j \in V_j$ be two vertices which are incident with at most one edge of $\mathcal{P}^i$ and $\mathcal{P}^j$, respectively. If $|\mathcal{P}^i \cup \mathcal{P}^j| \leqslant 2n-2$, then there exists a vertex $w^i \in V^i \setminus \{x^i, y^i\}$ such that $\{x^i, w^i\}$ and $\mathcal{P}^i$ are compatible and $\{w^j, y^j\}$ and $\mathcal{P}^j$ are compatible.*

3. Original proof and counterexample

In [5], the authors proved Lemma 4.7 (Lemma 1.1 in this note) case by case. The proofs of the cases except for Case 2.4 of this lemma are correct. The scenario of Case 2.4 is that $k \in \{0,1,2\} \setminus \{i,j\},\ e = (x^i, x^j)$ and $\{x^j, v\}$ and $\mathcal{P}^j$ are not compatible. The proof of Case 2.4 in [5] is as follows.

Proof. Case 2.4. $|\mathcal{P}^j| = 2n-4$.

In this case, $|\mathcal{P}^k| \leqslant 1$ and $|\mathcal{P}^i| \leqslant 1$. Since v has at least $2n-2 \geqslant 4$ neighbours in $Q[j]$, we may choose a neighbour w^j of v such that w^k is not incident with the edge in $\mathcal{P}^k$. By the induction hypothesis, $Q[j]$ has a Hamiltonian path $\pi[v, w^j]$ passing through $\mathcal{P}^j$ ... □

In the above proof, the authors tried to find a neighbour w^j of v such that w^k is not incident with the edge in $\mathcal{P}^k$ and use the induction hypothesis to construct a Hamiltonian path between v and w^j passing through $\mathcal{P}^j$ in $Q[j]$. In the following, we give an example in which the induction hypothesis fails to construct such a Hamiltonian path in $Q[j]$ because v and w^j does not satisfy the preconditions for the induction hypothesis.

Example 3.1. Let $\mathcal{P} = \{(001, 011), (001, 021), (110, 111), (221, 021)\}$ be a set of prescribed edges and $u = 211,\ v = 011$ be two vertices in Q_3^3. Let $Q[0],\ Q[1]$ and $Q[2]$ be a partition of Q_3^3 obtained by deleting all the edges in dimension 2 (see Fig. 1).

In Example 3.1, $|\mathcal{P}| = 4 = 2 \times 3 - 2$ and $\{u, v\}$ and $\mathcal{P}$ are compatible, $e = (221, 021) \in \mathcal{P} \setminus (\mathcal{P}^0 \cup \mathcal{P}^1 \cup \mathcal{P}^2)$, $|\mathcal{P}^0| = 2 = 2 \times 3 - 4,\ |\mathcal{P}^1| = 1$ and $\mathcal{P}^2 = \emptyset$ (see Fig. 1, the bold edges are prescribed edges). Clearly, $\{021, 011\}$ and $\mathcal{P}^0$ are not compatible. There are two neighbors 001 and 021 of v in $Q[0]$ such that their corresponding vertices are not incident with

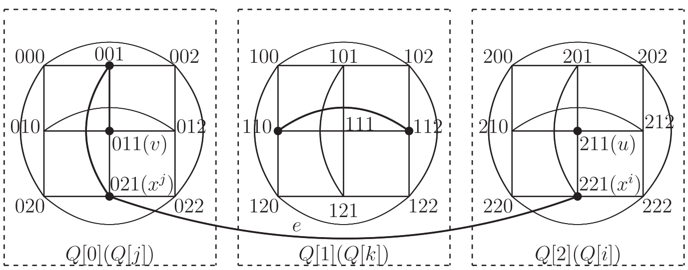

Fig. 1. The 3-ary 3-cube used in Example 3.1.