Information Sciences 296 (2015) 46–60

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Greedy randomized adaptive search procedure with exterior path relinking for differential dispersion minimization ☆

Abraham Duarte [a,*], Jesús Sánchez-Oro [a], Mauricio G.C. Resende [b], Fred Glover [c], Rafael Martí [d]

[a] Departamento de Ciencias de la Computación, Universidad Rey Juan Carlos, Spain
[b] Network Evolution Research Department, AT&T Labs Research, 200 S. Laurel Avenue, Room A5-1F34, Middletown, NJ 07748, USA
[c] OptTek Systems, Inc., Boulder, CO, USA
[d] Departamento de Estadística e Investigación Operativa, Universidad de Valencia, Spain

ARTICLE INFO

Article history:
Received 3 April 2014
Received in revised form 5 September 2014
Accepted 6 October 2014
Available online 30 October 2014

Keywords:
Dispersion problems
GRASP
Variable neighborhood search
Path relinking
Path separation

ABSTRACT

We propose several new hybrid heuristics for the differential dispersion problem, the best of which consists of a GRASP with sampled greedy construction with variable neighborhood search for local improvement. The heuristic maintains an elite set of high-quality solutions throughout the search. After a fixed number of GRASP iterations, exterior path relinking is applied between all pairs of elite set solutions and the best solution found is returned. Exterior path relinking, or *path separation*, a variant of the more common interior path relinking, is first applied in this paper. In interior path relinking, paths in the neighborhood solution space connecting good solutions are explored between these solutions in the search for improvements. Exterior path relinking, as opposed to exploring paths between pairs of solutions, explores paths beyond those solutions. This is accomplished by considering an initiating solution and a guiding solution and introducing in the initiating solution attributes not present in the guiding solution. To complete the process, the roles of initiating and guiding solutions are exchanged. Extensive computational experiments on 190 instances from the literature demonstrate the competitiveness of this algorithm.

1. Introduction

Equity problems are a family of NP-hard optimization problems that are an actual concern in the context of facility location, where the fairness among candidate facility locations is as relevant as the dispersion of the selected locations [36]. Given a set of elements, the main objective of these problems is to find a subset of those elements that minimizes a similarity measure. These kinds of problems have also applications in the context of urban public facility location [36], selection of homogeneous groups [3], dense/regular subgraph identification [20], and equity-based measures in network flow problems [4]. In spite of all these applications, most of the previous works are focused on the opposite family of problems, the dispersion problems, in which the objective is to maximize the differences among the selected elements. The dispersion problems have been widely studied. See Duarte and Martí [7] for a recent review of this problem. However, we have identified only one

☆ AT&T Labs Research Technical Report.
* Corresponding author.
E-mail addresses: abraham.duarte@urjc.es (A. Duarte), jesus.sanchezoro@urjc.es (J. Sánchez-Oro), mgcr@research.att.com (M.G.C. Resende), glover@opttek.com (F. Glover), rafael.marti@uv.es (R. Martí).

http://dx.doi.org/10.1016/j.ins.2014.10.010

previous metaheuristic-based paper on equitable problems, in which Prokopyev et al. [29] adapt a simple generic GRASP algorithm to solve several equitable problems.

Let $G = (V, E)$ be an undirected complete graph, where V is the set of n vertices and E the set of $\binom{n}{2}$ edges. Each edge $(u, v) \in E$ with $u, v \in V$ has an associated distance d_{uv} between u and v. Dispersion, or diversity, problems (DP) consist in finding a subset $S \subseteq V$ with m elements, such that an objective function (based on the distances between elements in S) is maximized or minimized. According to Prokopyev et al. [29], the objective of a dispersion problem can be either to identify a subset with (i) maximum distance among its elements (diversity problems), or (ii) with maximum similarity among them (equity problems). The first class of problems has been intensively studied in the last ten years. For instance, Martí et al. [24,25] and Gallego et al. [12] present several exact, heuristic, and metaheuristic-based methods for the maximum diversity problem. Two important variants are, respectively, the sum (Maxsum DP) and minimum (Maxmin DP) of the distances in the selected set [1].

Prokopyev et al. [29] propose four distinct equity-based functions to balance the diversity among the selected elements: the mean-dispersion function minimizes the average dispersion of the selected elements; the generalized mean-dispersion function, which is an extension of the mean-dispersion function, considers vertex-weighted graphs; and the min-sum and the min-diff dispersion functions that consider the extreme equity values of the selected elements. In this paper we focus on the last function, whose associated optimization problem is referred to as the *Minimum Differential Dispersion Problem* (Min-Diff DP). The Min-Diff DP is strongly NP-hard, and it remains NP-hard even if sign restrictions for distances between vertices are imposed [29]. Therefore, heuristic procedures emerges as the best option to obtain high quality solutions in shorter computing time.

A feasible solution of the Min-Diff problem is a set $S \subseteq V$ of m elements, where m is a given input parameter. Each feasible solution has associated with it a cost which can be computed as follows. Let $\Delta(v)$ be the sum of distances between a vertex $v \in S$ and the remaining elements of S. Formally,

$$\Delta(v) = \sum_{u \in S} d_{uv}.$$

The objective function of a solution S, denoted by $diff(S)$, is then computed as

$$diff(S) = \max_{u \in S} \Delta(u) - \min_{v \in S} \Delta(v).$$

Therefore, the Min-Diff problem consists of finding a solution $S^\star \subseteq V$ with the minimum differential dispersion, i.e.

$$S^\star = \arg\min_{S \subseteq V_m} diff(S),$$

where V_m is the set of all subsets of vertices in V with cardinality m.

Fig. 1a shows an example of a graph with six vertices and 15 edges with their associated distances. Fig. 1b and c depict two possible solutions for the Min-Diff problem for $m = 4$. The selected vertices in the solution are shown in black while the edges in each solution are highlighted by solid lines. The vertices not in the solution are shown in gray while the edges not in the solution are dashed. To evaluate the quality of each solution, we first compute the $\Delta(v)$ value for all the elements in the solution. In particular, Fig. 1b shows a solution where $S = \{A, B, D, E\}$, $\Delta(A) = 3 + 12 + 8 = 23$, $\Delta(B) = 3 + 3 + 2 = 8$, $\Delta(D) = 12 + 3 + 6 = 21$, and $\Delta(E) = 8 + 2 + 6 = 16$. The *diff*-value is calculated by first selecting the vertices having the highest and lowest Δ-values and then taking the difference of their Δ-values. In this solution, these vertices are, respectively, A and B, and therefore $diff(S) = \Delta(A) - \Delta(B) = 23 - 8 = 15$. If we now consider the solution $S' = \{A, C, E, F\}$ in Fig. 1c, it is easy to verify that the associated objective function value is $diff(S') = 8$. Considering that the Min-Diff problem is a minimization problem, solution S' is better than solution S. The rationale behind this is that the distances among the elements in S' are more similar than those among the elements in S.

Prokopyev et al. [29] present a basic mixed linear $0-1$ formulation of the problem. Let L_i and U_i be lower and upper bounds on the value of $\sum_{j \in S} d_{ij}$, i.e. $L_i = \sum_{j \in S} \min\{d_{ij}, 0\}$ and $U_i = \sum_{j \in S} \max\{d_{ij}, 0\}$. Then, the mixed linear $0-1$ formulation of the Min-Diff DP is as follows:

$$\begin{aligned}
\min_{t,r,s,x} \quad & t \\
s.t. \quad & t \geqslant r - s, \quad i = 1, \ldots, n \\
& r \geqslant \sum_{j:j \neq i} d_{ij} x_j - U_i(1 - x_i) + M^-(1 - x_i), \quad i = 1, \ldots, n \\
& s \leqslant \sum_{j:j \neq i} d_{ij} x_j - L_i(1 - x_i) + M^+(1 - x_i), \quad i = 1, \ldots, n \\
& \sum_{i=1}^{n} x_i = m \quad x \in \{0, 1\}^n,
\end{aligned}$$

where M^+ is an upper bound on the U_i values, M^- is a lower bound on the L_i values, and the binary decision variable $x_i = 1$ if and only if node $i \in S$.