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On distribution function of the diameter in uncertain graph

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ABSTRACT

In uncertain graphs, the existence of some edges is not predetermined. The diameter of an uncertain graph is essentially an uncertain variable, which indicates the suitability for investigation of its distribution function. The main focus of this paper is to propose an algorithm to determine the distribution function of the diameter of an uncertain graph. We first discuss the characteristics of the uncertain diameter, and the distribution function is derived. An efficient algorithm is designed based on Floyd's algorithm. Further, some numerical examples are illustrated to show the efficiency and application of the algorithm.

1. Introduction

To the best of our knowledge, in the classical graph theory, the edges and vertices are predetermined. Theoretical problems on graph theory are concerned with connectivity, nature of the graph and determination of diameter. To solve these problems, a variety of efficient algorithms have been proposed over the last decades and successfully applied to many real-world problems, such as transportation, communications, and supply chain management. More extensive information on classical graph theory can be found in [3,35].

In practice, indeterminacy is inevitable due to the lack of information. The classical algorithms are appear to be very difficult to apply directly the indeterminacy in respect of vertices and edges. In this paper, we have studied the existence of some nondeterministic edges. The existence of such nondeterministic graphs are used to represent relationship network of a group. As an example, Fig. 1 depicts "confirmed friends" which represents completely confirmed relationships and "potential friends" represents relationships that are inferred from other information, such as personal data and friends list. In other words, in a nondeterministic graph describing relationship network, "potential friends" are represented by nondeterministic edges. It is shown how an existence chance α_{ij} is associated to and nondeterministic edge e_{ij}.

To deal with nondeterministic graphs, some researchers introduced probability theory and developed random graphs. Erdös and Réyi [9,10] first used probability theory for nondeterministic graphs. Moreover, Gilbert [12] in 1959 dealt independently E–R random graphs. Usually, an E-R random graph is obtained by starting with a set of n isolated vertices and adding successive edges between them with probability $0 < p < 1$. In 1998, Watts and Strogatz [34] proposed the concept of "small world" network between the deterministic graph and E–R random graph, which emphasized the cluster phenomenon of social networks. Similar researches can also be found in literature [24,25,30]. The motivation of these researches leads to qualitatively explain phenomena in networks, and the probability is more like a theoretical parameter.

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As in relationship networks, the existence of chance of edges is sometimes evaluated by experts, based on which the structure of the nondeterministic graph is analyzed and decisions are then made. It has been emphasized by [20,33] that the expert data tend to give too much weight on unlikely events. In other words, it is unsuitable to use probability to handle nondeterministic information with expert data. Motivated by this point, some non-probabilistic methods have been proposed and developed since 1960s, such as fuzzy mathematics (Dubois and Prade [8], Zadeh [37,38]), rough sets theory(Gong et al. [17], Pawlak [26]), evidence theory (Dempster [5]), and methods of Granular Computing (Pedrycz [27], Bargiela and Pedrycz [1]). These methods are also employed to handle nondeterministic graphs with expert data, for instance fuzzy graphs (Bhattacharya [2], Rosenfeld [29]) and rough graphs (Chen and Li [4], He et al. [19]).

Uncertainty theory, proposed in 2007 and refined in 2010 by Liu [21,23], is another efficient tool to handle nondeterministic information with expert data. In recent years, uncertainty theory has been widely used in the field of operational research, such as network optimization [14,16,18,39], inventory problem [28,15,6,7] and transportation problem [31,32,36]. In 2013, Gao and Gao [13] first introduced uncertainty theory into graph theory, and proposed a concept of uncertain graph, in which the existence of chance of edges is described by uncertain measure. In their paper, the connectedness index, which measures the chance that an uncertain graph is connected, was first proposed. To calculate the connectedness index, two algorithms were designed, which were based on the Kruskal's algorithm and Prim's algorithm respectively.

To the best of our knowledge, so far there is currently no related study on the diameter of uncertain graphs in literature. In view of this fact, this paper focuses on investigating the diameter of uncertain graphs. Note that the diameter of uncertain graph is an uncertain variable due to the existence of uncertain edges. It is thus an meaningful issue to explore how to formulate and compute the distribution function of the diameter in an uncertain graph. In this paper, we first study the characteristics of diameter in an uncertain graph, and then obtain the corresponding distribution function. Moreover, we explicitly design an algorithm derived from the Floyd's algorithm [11] to calculate the distribution function. The efficiency of the algorithm is finally shown theoretically and experimentally.

The remainder of this paper is organized as follows. In Section 2, uncertainty theory is introduced briefly for the completeness of this research. Section 3 gives the problem descriptions and defines the concept of the diameter in uncertain graphs. In Section 4, the distribution function of the diameter of uncertain graph is obtained. In Section 5, an efficient algorithm for calculating the distribution function is proposed and illustrated with some numerical examples. Section 6 concludes this paper with a brief summary.

2. Uncertainty Theory

For the completeness of this study, we introduce here some basic properties of uncertainty theory.

Let Γ be a nonempty set, and $\mathcal{L}$ a σ-algebra over Γ. Each element $\Lambda \in \mathcal{L}$ is assigned a number $\mathcal{M}\{\Lambda\}$. In order to ensure that the number $\mathcal{M}\{\Lambda\}$ has certain mathematical properties, Liu [21] presented the following three axioms:

Axiom 1 (*Normality*). $\mathcal{M}\{\Gamma\} = 1$.

Axiom 2 (*Duality*). $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$ for any event Λ.

Axiom 3 (*Subadditivity*). For every countable sequence of events $\{\Lambda_i\}$, we have

$$\mathcal{M}\left\{\bigcup_{i=1}^{\infty}\Lambda_i\right\} \leqslant \sum_{i=1}^{\infty}\mathcal{M}\{\Lambda_i\}.$$

Definition 1 (*Liu* [21]). The set function $\mathcal{M}$ is called an uncertain measure if it satisfies the normality, self-duality, and countable subadditivity axioms. The triplet $(\Gamma, \mathcal{L}, \mathcal{M})$ is called uncertain space.

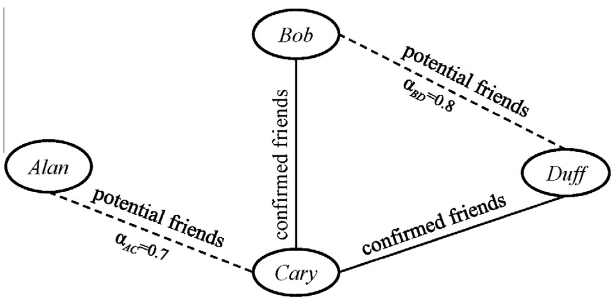

Fig. 1. A relationship network with potential relationships.