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A fractional-order adaptive regularization primal–dual algorithm for image denoising

Dan Tian [a,b], Dingyu Xue [a], Dianhui Wang [c,*]

[a] Department of Information Science and Engineering, Northeastern University, Shenyang, Liaoning 110004, China
[b] Department of Information Engineering, Shenyang University, Shenyang, Liaoning 110044, China
[c] Department of Computer Science and Computer Engineering, La Trobe University, Melbourne, Victoria 3086, Australia

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A B S T R A C T

This paper aims to develop a fractional-order model and a primal–dual algorithm for image denoising, where a regularization parameter can be adjusted adaptively according to Morozov discrepancy principle at each iteration to ensure that the denoised image retains in a specific set. In the light of saddle-point theory, the convergence of our proposed algorithm is guaranteed. Simulations with comparisons are carried out to demonstrate the effectiveness of our proposed algorithm for image denoising.

1. Introduction

Image denoising techniques remove noises from images based on various models and algorithms. Mathematically, this task can be formulated as an estimate problem, i.e., finding an approximation of an original image u from a degraded or noisy image g:

$$g = u + n, \tag{1}$$

where n represents the Gaussian noise. In the past decades, a number of methods have been developed to estimate u, including wavelet transforms [1], partial differential equations [2], Fourier transforms [3], and total variation approaches [4]. Among these denoising algorithms, the total variation method is quite effective in removing noises while preserving edges, which specifies n as zero-mean Gaussian white noise with variance σ^2. One of the most well known total variation models was introduced by Rudin, Osher and Fatemi (ROF), which formulated the denoising problem as a minimization task [5], that is,

$$\min_{u \in X} \|\nabla u\|_1 + \frac{\lambda}{2}\|u - g\|_2^2, \tag{2}$$

where X is a finite dimensional vector space, $\|\bullet\|_v$ represents a $v-$ norm, ∇ is a gradient operator, and λ is a regularizing factor.

* Corresponding author.
E-mail addresses: www.sltd2008@163.com (D. Tian), xuedingyu@ise.neu.edu.cn (D. Xue), dh.wang@latrobe.edu.au (D. Wang).

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The first term in (2) is usually called as a regularization term, which is used in the ROF model and responsible for regulating the sharpness of edges. The second term is called as a fidelity term which contributes to control the degree of approximation accuracy. However, the ROF model favors a piecewise constant solution which often leads to the blocky effect, and small details are often filtered out in the process of denoising. To solve this problem, fractional-order differential was introduced to the total variation denoising models because the fractional-order differential takes more neighboring pixel information into account. In [6], a class of fractional-order anisotropic diffusion models was proposed for noise removal, where a discrete Fourier transform was employed to evaluate the fractional differential values. In [7], a fractional-order multi-scale variation model was suggested for problem solving and an alternating projection algorithm was developed to resolve the model. In [8], a fractional-order TV-L2 model was developed for image denoising and the majorization–minimization (MM) algorithm [9] was used to decompose the problem into a set of linear optimization problems, which can be solved by using the conjugate gradient algorithm. In [10], a class of fractional-order variational models was presented and the solution was provided by using the gradient descent method. Based on the results reported in the publications mentioned above, it can be confirmed that the fractional differential regularization model performs favorably for image denoising in terms of less blocky effect and detail preservation.

There are many numerical algorithms for solving the variation problem related to image denoising. However, they suffer from slow running or high computational complexity or lack of knowledge on the selection of the regularization parameter. In this paper, by referring the primal–dual algorithm [11] for solving saddle-point problems, we propose a primal–dual formulation to solve the fractional-order ROF model. To balance the edge preservation and the fidelity properly, we employ the Morozov discrepancy principle [12] to adaptively adjust the regularization parameter so that the resulting denoised images will satisfy certain constraints.

The remainder of this paper is organized as follows: Section 2 describes a fractional-order primal–dual model for image denoising. Section 3 details the proposed algorithm with a convergence analysis and a tuning scheme of the regularization parameter. Section 4 reports our simulation results with comparisons, and Section 5 concludes this work.

2. Fractional-order denoising models

Fractional-order differential has been applied in image processing although its definition is not unified up to date. In literature, the definition given by Grünwald–Letnikov (G–L) and Riemann–Liouville (R–L) are popular and widely used in image processing [13]. In this paper, we adopt G–L's definition in problem solving. Let the size of an image u be $M \times N$. Thus, the discrete form of the fractional-order gradient $\nabla^\alpha u$ can be evaluated by

$$(\nabla^\alpha u)_{i,j} = \left((\Delta_1^\alpha u)_{i,j}, (\Delta_2^\alpha u)_{i,j} \right), \tag{3}$$

with $1 \leqslant i \leqslant M, 1 \leqslant j \leqslant N$, and

$$\begin{cases} (\Delta_1^\alpha u)_{i,j} = \sum_{k=0}^{K-1} (-1)^k C_k^\alpha u_{i-k,j}, \\ (\Delta_2^\alpha u)_{i,j} = \sum_{k=0}^{K-1} (-1)^k C_k^\alpha u_{i,j-k}, \end{cases} \tag{4}$$

where $K \geqslant 3$ is an integer constant, $C_k^\alpha = \frac{\Gamma(\alpha+1)}{\Gamma(k+1)\Gamma(\alpha-k+1)}$, and Γ is the gamma function.

Based on the variation model of integer order, we generalize a fractional-order ROF model as follows:

$$\min_{u \in X} \|\nabla^\alpha u\|_1 + \frac{\lambda}{2} \|u - g\|_2^2, \tag{5}$$

where

$$\begin{cases} \|\nabla^\alpha u\|_1 = \sum_{i,j} \left| (\nabla^\alpha u)_{i,j} \right|, \\ \left| (\nabla^\alpha u)_{i,j} \right| = \sqrt{((\Delta_1^\alpha u)_{i,j})^2 + \left((\Delta_2^\alpha u)_{i,j} \right)^2}. \end{cases} \tag{6}$$

Note that the fractional differential in (4) is a global operator and useful to preserve more image details. Theoretically, the number of these terms may be very large, but one usually takes $K = min\{M, N\}$ for image processing. While applying the well-known Euler–Lagrange method to resolve model (5), we need to compute a derivative of a non-differentiable function (i.e., the fractional order regulation term). This turns us to find an alternative solution by considering a primal–dual model. We first introduce some notation used in the dual ROF model.

Let $p_{i,j} = \left(p_{i,j}^1, p_{i,j}^2\right), |p_{i,j}| = \sqrt{\left(p_{i,j}^1\right)^2 + \left(p_{i,j}^2\right)^2} \leqslant 1$, and Y represent the set of all $p_{i,j}$. Then, we have

$$\|\nabla^\alpha u\|_1 = \sup_{p \in Y} < \nabla^\alpha u, p > . \tag{7}$$