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On sampled-data fuzzy control design approach for T–S model-based fuzzy systems by using discretization approach ☆

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In this paper, a sampled-data fuzzy control design approach for Takagi–Sugeno (T–S) model-based fuzzy systems is proposed. Firstly, the T–S model-based fuzzy systems are approximated by T–S fuzzy model-based discrete systems with parameter uncertainties. Then a sufficient condition on the existence of a sampled-data fuzzy controller for the fuzzy systems is formulated in the form of linear matrix inequalities which are proposed in some symmetrical form. So the proposed fuzzy control design approach will be less conservative. And some problems in the existing literature can be solved. Finally, a simulation example is discussed to show the effectiveness of the obtained fuzzy control design approach in this paper.

1. Introduction

A fuzzy control system combines information of human experts (natural language) with measurements and mathematical models. Recently, there has been an increasing interest in the stability analysis and controller design for nonlinear systems. Fuzzy systems can offer more advantages in handling some nonlinear systems. There are two different approaches to fuzzy systems design: Type-1 fuzzy systems and Type-2 fuzzy systems. The latter is proposed as an extension of the former with the intention of being able to model the uncertainties that are invariably existent in the fuzzy systems [9]. More results can be found on Type-1 and Type-2 fuzzy logic approaches [1–3,8,12]. Since Takagi and Sugeno [13] considered the fuzzy control design problem for nonlinear systems via linear T–S fuzzy models, there has been an increasing interest in fuzzy logic control based on the T–S model for nonlinear systems. T–S model approach offers a powerful and systematical control methodology to handle nonlinear systems. It gives an effective way to represent complex nonlinear systems by some simple local linear dynamic systems, and some analysis and design methods of the linear systems can be effectively extended to the T–S fuzzy systems [14]. During the past two decades, considerable attention has been paid on the fuzzy systems in the form of the Takagi–Sugeno (T–S) models [5–7,11,16–19].

Owing to the rapid growth of the digital circuit technologies, digital controller is generally utilized for some complex dynamical systems [10,11]. Usually, a discrete-time control input signal is produced by a digital computer and then is further converted back into a continuous-time control input signal using a zero-order holder. In such a case, the overall system becomes a sampled-data system, where the control signals are kept to be a constant during the sampling period and are only

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allowed to change at the sampling instant [4–7,11,17–19]. Recently, much attention has been paid to the investigation of the sampled-data nonlinear systems in the form of the following T–S model-based fuzzy model.

$$\textbf{Plant Rule } R^i: \quad \text{IF } z_1(t) \text{ is } M_{i1},\ z_2(t) \text{ is } M_{i2},\dots,z_g(t) \text{ is } M_{ig} \\ \text{THEN } \begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t), \\ x(0) = x_0 \end{cases} \tag{1}$$

where $i = 1,2,\dots,r$ and r is the number of IF–THEN rules; $z_1(t),\ z_2(t),\dots,z_g(t)$ are the premise variables of (1) and $M_{ij}(i = 1,2,\dots,r;\ j = 1,2,\dots,g)$ are the fuzzy sets corresponding to $z_j(t)$ and the plant rules; $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are the state vector and input vector, respectively; x_0 is the initial condition of the system (1); A_i and B_i are known parameter matrices of appropriate dimensions. By using a center average defuzzifer, product inference, and singleton fuzzifier, the global dynamics of the T–S fuzzy system (1) is described by

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))[A_i x(t) + B_i u(t)], \tag{2}$$

where

$$h_i(z(t)) = \frac{\mu_i(z(t))}{\sum_{i=1}^{r}\mu_i(z(t))}, \quad \mu_i(z(t)) = \prod_{j=1}^{g} M_{ij}(z_j(t)) \\ z(t) = [z_1(t), z_2(t),\dots,z_g(t)]$$

in which $M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in M_{ij}. Then it can be found that

$$\mu_i(z(t)) \geqslant 0,\ (i = 1,2,\dots,r), \quad \sum_{i=1}^{r}\mu_i(z(t)) > 0$$

for all t. Therefore

$$h_i(z(t)) \geqslant 0,\ (i = 1,2,\dots,r), \quad \sum_{i=1}^{r} h_i(z(t)) = 1 \tag{3}$$

hold for all t. The following state feedback T–S fuzzy-model-based sampled-data control law will be employed for the system (2) by utilizing the idea of parallel distributed compensation (PDC) [15,16], in which the same fuzzy sets with the fuzzy model are shared for the designed fuzzy sampled controller in the premise parts

$$R^i: \quad \text{IF } z_1(kT) \text{ is } M_{i1},\ z_2(kT) \text{ is } M_{i2},\dots,z_g(kT) \text{ is } M_{ig} \\ \text{THEN } u_i(t) = K_i x(kT), t \in [kT,(k+1)T) \tag{4}$$

where $K_i\ (i = 1,2,\dots,r)$ are the controller gains of (4) to be determined, T is the sampling time, $x(kT)$ is the state vector at the instant kT by using a zero-order holder; $u_i(t)$ is the input vector of rule i. Analogous to (2), the defuzzified output of the controller rules is given by

$$u(t) = \sum_{i=1}^{r} h_i(z(kT))K_i x(kT),\ t \in [kT,(k+1)T). \tag{5}$$

Substituting (5) into (1) yields the closed-loop fuzzy system of (1)

$$\textbf{Plant Rule } R^i: \quad \text{IF } z_1(t) \text{ is } M_{i1},\ z_2(t) \text{ is } M_{i2},\dots,z_g(t) \text{ is } M_{ig} \\ \text{THEN } \begin{cases} \dot{x}(t) = \sum_{j=1}^{r} h_j(z(kT))[A_i x(t) + B_i K_j x(kT)], \\ x(0) = x_0 \end{cases} \tag{6}$$

for $t \in [kT,(k+1)T), k = 1,2,\dots$. The resulting global closed-loop system is represented by

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(kT))[A_i x(t) + B_i K_j x(kT)] \tag{7}$$

In order to obtain a sampled-data control design approach for the T–S model-based fuzzy system (1), the system (1) is often represented by the form of continuous-time systems with delayed control inputs [6,11,17]. In such a case, the digital control law is represented as delayed control between two sampling instants. For obtaining a less conservative result, one often estimates the derivative of the proposed Lyapunov–Krasovskii functional $V(x_t)$ as