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Image restoration using total variation with overlapping group sparsity

Jun Liu [a], Ting-Zhu Huang [a,*], Ivan W. Selesnick [b], Xiao-Guang Lv [c], Po-Yu Chen [b]

[a] School of Mathematical Sciences/Institute of Computational Science, University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, PR China
[b] Polytechnic Institute of New York University, Brooklyn, NY 11201, USA
[c] School of Science, Huaihai Institute of Technology, Lianyungang, Jiangsu 222005, PR China

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ABSTRACT

Image restoration is one of the most fundamental issues in imaging science. Total variation regularization is widely used in image restoration problems for its capability to preserve edges. In the literature, however, it is also well known for producing staircase artifacts. In this work we extend the total variation with overlapping group sparsity, which we previously developed for one dimension signal processing, to image restoration. A convex cost function is given and an efficient algorithm is proposed for solving the corresponding minimization problem. In the experiments, we compare our method with several state-of-the-art methods. The results illustrate the efficiency and effectiveness of the proposed method in terms of PSNR and computing time.

1. Introduction

Image restoration is one of the most fundamental issues in imaging science and plays an important role in many mid-level and high-level image processing applications. On account of the imperfection of an imaging system, a recorded image may be inevitably degraded during the process of image capture, transmission, and storage. In this paper, we consider a common degradation model as the following linear system

$$g = Hf + \eta, \tag{1}$$

where g represent the blurred and noisy observation, f is the desired true image, and η is Gaussian white noise with zero mean. H is a blurring matrix constructed from the discrete point spread function, together with the given boundary conditions. Unless stated otherwise, we assume that the underlying images have square domains of size $n \times n$ and let $g,\ f$ and η be the n^2-length vectors by lexicographically ordering the two-dimensional images, respectively.

It is well known that image restoration belongs to a general class of problems which are rigorously classified as ill-posed problems [32,50]. To tackle the ill-posed nature of the problems, regularization techniques are usually used to obtain a stable and accurate solution [30]. In other words, we seek to estimate the original image f by solving the following variational problem:

$$\min_f \left\{ \frac{1}{2}\|g - Hf\|_2^2 + \lambda\varphi(f) \right\}, \tag{2}$$

* Corresponding author.
E-mail addresses: junliucd@163.com (J. Liu), tingzhuhuang@126.com (T.-Z. Huang).

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where $\|\cdot\|_2$ denotes the Euclidean norm, $\varphi(f)$ is conventionally called a regularization functional, and $\lambda > 0$ is referred to as a regularization parameter which controls the balance between fidelity term and regularization term in (2).

How to choose a good regularization functional $\varphi(f)$ is an active area of research in imaging science. In the early 1960s, Phillips [44] and Tikhonov [49] proposed the definition of φ as an l_2-type norm (often called Tikhonov regularization in the literature), that is, $\varphi(f) = \|Lf\|_2^2$ with L an identity operator or difference operator. The functional $\varphi(f)$ of this type has the advantage of simple calculations, however, it overly smoothes edges which are important features in human perception. Therefore, it is not a good choice since natural images always have many edges. To overcome this shortcoming, Rudin et al. [45] proposed using the total variation (TV) seminorm to replace the l_2-type norm, that is, they set $\varphi(f) = \|\nabla f\|_1$. Then the corresponding minimization problem is

$$\min_f \left\{ \frac{1}{2}\|g - Hf\|_2^2 + \lambda \|\nabla f\|_1 \right\}, \tag{3}$$

where $\|\nabla f\|_1 = \sum_{i,j=1}^n \left\|(\nabla f)_{i,j}\right\|$ and the discrete gradient operator $\nabla : \mathbb{R}^{n^2} \to \mathbb{R}^{2\times n^2}$ is defined by $(\nabla f)_{i,j} = ((D^{(1)}f)_{i,j}, (D^{(2)}f)_{i,j})$ with

$$((D^{(1)}f)_{i,j} = \begin{cases} f_{i+1,j} - f_{i,j} & \text{if} \quad i < n, \\ f_{1,j} - f_{n,j} & \text{if} \quad i = n, \end{cases}$$

and

$$((D^{(2)}f)_{i,j} = \begin{cases} f_{i,j+1} - f_{i,j} & \text{if} \quad j < n, \\ f_{i,1} - f_{i,n} & \text{if} \quad j = n, \end{cases}$$

for $i,j = 1, 2, \ldots, n$. Note that $f_{i,j}$ refers to the $((j-1)n + i)$th entry of the vector f (it is the (i,j)th pixel location of the $n \times n$ image, and this notation is valid throughout the paper). Here, $D^{(1)}$ and $D^{(2)}$ are $n^2 \times n^2$ the first-order finite difference matrices in the horizontal and vertical directions, respectively. The problem (3) is commonly referred to as the ROF model. The TV is isotropic if the norm $\|\cdot\|$ is the Euclidean norm and anisotropic if l_1-norm is defined.

In the literature, many algorithms have been proposed for solving the ROF model (3). If H is the identity matrix, then the problem (3) is referred to as the denoising problem. In the pioneering work [45], the authors proposed to employ a time marching scheme to solve the associated Euler–Lagrange equation of (3). However, their method was very slow due to CFL stability constraints [48]. Chambolle [6] studied a dual formulation of the TV denoising problem and proposed a semi-implicit gradient descent algorithm to solve the resulting constrained optimization problem. He also proved his algorithm was globally convergent with a suitable step size. In [25], Goldstein and Osher proposed the novel split Bregman iterative algorithm to deal with the artificial constraints, their method had several advantages such as fast convergence rate and stability. If H is a blurring operator, the model (3) is then related to the image deblurring problem. Wang and Yang et al. [51] proposed a fast total variation deconvolution (FTVd) method based on splitting technique and constructed an iterative procedure of alternately solving a pair of easy subproblems associated with an increasing sequence of penalty parameter values. Their approach belongs to penalty methods from the perspective of optimization. In [1], Beck and Teboulle studied a fast iterative shrinkage-threshold algorithm (FISTA) which was a non-smooth variant of Nesterov's optimal gradient-based algorithm for smooth convex problems [41]. More recently, Chan et al. [9] proposed an efficient and effective method by imposing box constraint on the ROF model (3). Their numerical experiments showed that their method could obtain much more accurate solutions and was superior to some state-of-the-art algorithms for unconstrained models. If H is not known, the problem (3) is then the commonly known as blind deblurring which is out of scope in this paper, we refer the reader to see [26] and the related references therein.

Although TV regularization has been proved to be extremely useful in a variety of applications, it yields staircase artifacts [10,16]. Therefore, the approaches involving the classical TV regularization often develop false edges that do not exist in the true image since they tend to transform smooth regions (ramps) into piecewise constant regions (stairs). To avoid these drawbacks, there is a growing interest for replacing the TV regularizer by the high order TV regularizer, which can comprise more than merely piecewise constant regions. The majority of the high order norms involve second order differential operators because piecewise-vanishing second order derivatives lead to piecewise-linear solutions that better fit smooth intensity changes [34]. In [36], the authors chose the regularization functional $\varphi(f) = \left\|\nabla^2 f\right\|_1$. Then the minimization problem (2) is treated as the following high order TV based scheme:

$$\min_f \left\{ \frac{1}{2}\|g - Hf\|_2^2 + \lambda \left\|\nabla^2 f\right\|_1 \right\}, \tag{4}$$

where $\left\|\nabla^2 f\right\|_1 = \sum_{i,j=1}^n \left\|(\nabla^2 f)_{i,j}\right\|_2$ with $(\nabla^2 f)_{i,j} = \left((D_{xx}f)_{i,j}, (D_{yx}f)_{i,j}; (D_{xy}f)_{i,j}, (D_{yy}f)_{i,j}\right)$, for $i,j = 1, 2, \ldots, n$. Here $(D_{st}f)_{i,j}, s,t \in \{x,y\}$ denotes the second order difference of f at pixel (i,j).

In our previous work [46], we used group sparsity concepts for the one dimension signal denoising problem. We showed that, for general signal denoising and restoration, the groups (clusters) of large values may arise anywhere in the domain of the signal. In this case, if the group structure was defined a priori, a group of large values may straddle two of the predefined groups. Hence, it is suitable to formulate the problem in terms of overlapping groups. Then we utilized the overlapping group