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Likelihoods of interval type-2 trapezoidal fuzzy preference relations and their application to multiple criteria decision analysis

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ABSTRACT

Interval type-2 fuzzy sets are useful and valuable for depicting uncertainty and managing imprecision in decision information. In particular, interval type-2 trapezoidal fuzzy numbers, as a special case of interval type-2 fuzzy sets, can efficiently express qualitative evaluations or assessments. In this work, the concept of the likelihoods of interval type-2 trapezoidal fuzzy preference relations based on lower and upper likelihoods is investigated, and the relevant properties are discussed. This paper focuses on the use of likelihoods in addressing multiple criteria decision analysis problems in which the evaluative ratings of the alternatives and the importance weights of the criteria are expressed as interval type-2 trapezoidal fuzzy numbers. A new likelihood-based decision-making method is developed using the useful concepts of likelihood-based performance indices, likelihood-based comprehensive evaluation values, and signed distance-based evaluation values. A simplified version of the proposed method is also provided to adapt the decision-making context in which the importance weights of the criteria take the form of ordinary numbers. The practical effectiveness of the proposed method is validated with four applications, and several comparative analyses are conducted to verify the advantages of the proposed method over other multiple criteria decision-making methods.

1. Introduction

Multiple criteria decision analysis (MCDA) addresses the ranking of alternatives and the selection of the best of a finite set of alternatives based on a finite set of criteria [41]. Uncertain and imprecise assessments of information commonly occur in practical MCDA situations [24,47], particularly with respect to a lack of knowledge or experience, intangible or non-monetary criteria, or a complex and uncertain environment [10]. Therefore, modeling uncertainty in subjective human management becomes increasingly important in handling MCDA problems [33,46]. Interval type-2 fuzzy sets are useful for depicting uncertainty and managing imprecision in MCDA.

Interval type-2 fuzzy sets are the most widely used type of type-2 fuzzy sets [11,12,39,49]. In real-world situations, it is often not justifiable or technically viable to quantify grades of membership in terms of a single numeric value, which gives rise to the idea of type-2 fuzzy sets [40]. The concept of type-2 fuzzy sets, which was initially introduced by Zadeh [45], is an

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extension of type-1 fuzzy sets, in which the membership function falls into a fuzzy set on the interval [0, 1] [28,48]. Type-1 fuzzy sets cannot fully handle all of the uncertainty that is present in real-world problems [25,38] because it is not reasonable to use an accurate membership function for an uncertain item [6,36]. In contrast, type-2 fuzzy sets can model second-order uncertainties [14,23]. If all of the second-order uncertainties disappear, type-2 fuzzy logic will reduce to type-1 fuzzy logic [25,26]. Fuzzy set theory can provide a systematic calculus to address linguistic information that improves the numerical computations by using linguistic labels that are stipulated by membership functions [37]. Type-2 fuzzy sets are better than type-1 fuzzy sets for handling linguistic uncertainties by modeling vagueness and unreliability of information [6,7,36] and for simulating language evolution by type-2 fuzzy grammar [2,3].

Although type-2 fuzzy sets can be used in cases with a high degree of uncertainty, they may require an undesirably large amount of computation [28]. The computational complexity of using type-2 fuzzy sets is significantly high, which makes them difficult to employ in the real world [9,48,50]. Therefore, considerable interest has arisen about interval type-2 fuzzy sets because their membership values take the form of intervals [40] (i.e., the secondary membership (grade) is equal to 1), which makes the associated computations manageable. Interval type-2 fuzzy sets have been successfully applied to MCDA problems [1,8–10,22,27,39]. In particular, numerous studies have been conducted on MCDA methods with interval type-2 trapezoidal fuzzy (IT2TrF) numbers [5,9,11–16,43,49]. IT2TrF numbers can efficiently express linguistic evaluations or assessments by objectively transforming linguistic terms into numerical variables [13,49].

However, most existing MCDA methods that are based on IT2TrF numbers within the interval type-2 fuzzy environment require complicated and sophisticated computations. For example, Chen [15] developed an interval type-2 fuzzy preference ranking organization method for enrichment evaluations (PROMETHEE) based on IT2TrF numbers. To use the PROMETHEE-based outranking method, decision makers and analysts must identify the appropriate signed distance-based generalized criteria (i.e., signed distance-based usual criterion, U-shaped criterion, V-shaped criterion, level criterion, V-shaped with indifference criterion, and Gaussian criterion) for each decision attribute and define the corresponding preference functions based on interval type-2 fuzzy sets. Next, the signed distance-based comprehensive preference indices must be derived by combining the criterion importance and the preference functions. To acquire the priority order of the alternatives, decision makers and analysts must determine the exiting flows, entering flows, and the net flows for partial and complete preordering. Using this newly developed method is a meticulous and complicated task for decision makers and analysts. Numerous existing MCDA methods that are based on interval type-2 fuzzy sets (e.g., the PROMETHEE-based outranking method) have solid theoretical bases and rigorous implementation procedures. However, the mathematical manipulations and computations associated with interval type-2 fuzzy data (e.g., IT2TrF numbers) are relatively complex and troublesome for MCDA processes in practice. Therefore, this paper attempts to provide a simple and effective method for processing sophisticated IT2TrF numbers in MCDA problems.

Considering the ease of employment and the computational efficiency, this paper develops a modified concept of likelihoods based on IT2TrF numbers and presents a new MCDA procedure that is simple and easy to implement for practical applications. Based on the IT2TrF framework, this paper describes the development of a new MCDA method that is based on the likelihoods of IT2TrF preference relations. Approaches that use likelihood-based comparisons have been applied to address multiple criteria decision-making problems [18–20,29–32,44]. This paper proposes a modified concept of likelihoods based on IT2TrF numbers that differs from previous approaches. By comparing a lower fuzzy set and an upper fuzzy set within two IT2TrF numbers, this paper determines a lower likelihood and an upper likelihood of the possibility of preference relations, which are different from the likelihoods introduced by Chen and Lee [19,20]. The likelihood of IT2TrF preference relations can be determined using the mean of the lower and upper likelihoods and can be used to establish a simple and useful likelihood-based MCDA method.

The purpose of this paper is to explore the properties of the likelihoods of IT2TrF preference relations and to develop a likelihood-based method for handling MCDA problems within the IT2TrF decision environment. While considering the context of IT2TrF numbers, this paper introduces an extended concept of likelihoods that originates from the fuzzy preference relations presented by Chen and Lee [18] and the likelihoods proposed by Lee and Chen [29] and Chen and Lee [19,20]. We develop the concepts of lower and upper likelihoods to derive the range of possibilities of IT2TrF preference relations and to subsequently determine the likelihoods of the IT2TrF numbers. Additionally, we investigate several useful and valuable properties that are relevant to the likelihoods of IT2TrF preference relations. Considering an MCDA problem based on IT2TrF numbers, we employ the likelihoods between the evaluative ratings to propose the concept of likelihood-based performance indices, and we incorporate the importance weights of the criteria to acquire a likelihood-based comprehensive evaluation value for each alternative. A signed distance-based evaluation value is established using the concept of signed distances, and the priority ranking of the alternatives can be subsequently determined according to the descending order of the signed distance-based evaluation values. Finally, the feasibility and applicability of the proposed likelihood-based MCDA method are illustrated using four MCDA problems, including the medical decision-making problem presented by Chen et al. [16], the car evaluation problem presented by Chen and Lee [19], the manager selection problem presented by Liu and Jin [34], and the car purchase problem presented by Chen and Wang [22]. Additionally, comparisons with different MCDA methods verify the effectiveness and advantages of the proposed method of using the likelihoods of IT2TrF preference relations.

The proposed likelihood-based MCDA method makes two main contributions. First, a modified concept of likelihoods that is based on IT2TrF numbers is constructed to investigate the range of possibilities of the IT2TrF preference relations. Second, a simple and effective MCDA procedure is developed using the useful concepts of likelihood-based performance indices, likelihood-based comprehensive evaluation values, and signed distance-based evaluation values. As opposed to previous