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Expressive efficiency of two kinds of specific CP-nets

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ABSTRACT

CP-nets (conditional preference networks) are a graphical model for compactly expressing conditional *ceteris paribus* (all other things being equal) preference statements on multi-attribute domains. In this paper, we investigate the expressive efficiency of two kinds of binary-valued CP-nets, the first kind is set-structured CP-nets, and the second kind is equal difference CP-nets. For the first kind, we prove that it can express $3^n - 2^n$ preference relations with n preference rules, and has an expressive efficiency of $(3^n - 2^n)/n$. For the second kind, we show that it can express a total order of $2^{n-1} * (2^n - 1)$ preference relations with $2^n - 1$ preference rules, and has an expressive efficiency of 2^{n-1}. For the future research, we propose an open problem: given an acyclic CP-net N with the in-degree sequence of $(d_1, d_2, \ldots, d_n)$, how many preference relations can be expressed by N?

1. Introduction

Preference information plays a crucial role in understanding and controlling effective reasoning, especially, it can guide human's behavior selections and decision makings in many scenarios [8]. For instance, in early childhood, when facing many toys, if our parents only prepared to buy one toy for us, the key problem is to decide which one is the most preferred on the basis of certain preference criteria. As the second example, in complex professional and organizational decisions, such as buying stocks, people have to think comprehensively whether a stock is preferred to the other one. In preference decisions, there are two types of preferences: quantitative preference (e.g., I prefer tea at level 0.6, and that I prefer coffee at level 0.4) and qualitative preference (e.g., I prefer coffee to tea). Qualitative preference is also called ordinal preference represented by binary relation [29], whereas quantitative preference is called cardinal preference represented by utility function [28]. Moreover, preferences can be unconditional, such as "I prefer fish wherever", or conditional, such as "if there is white wine, I prefer fish to meat". Generally speaking, expressing qualitative preference relation is rather easy, nevertheless, expressing corresponding quantitative preference value may be tedious or difficult for users owing to human cognitive difficulties [10]. Consequently, it is a natural way to allow users to express their preferences in a qualitative way.

Among the preference languages, CP-nets (conditional preference networks) are a graphical model for compactly representing conditional, qualitative preference relations [25,3]. In this paper, we investigate expressive efficiency of two kinds of CP-nets, i.e., set-structured and equal difference CP-nets. The remainder of the paper is structured as follows. Following the introduction to preference and CP-nets, Section 2 gives a sketch of the preference representation and our contributions in this paper. In Section 3, we present some basic concepts about the formalism and semantics of CP-nets as well as a fashionable example illustrating the CP-nets as a graphical model. Section 4 is concerned with the expressive efficiency of

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set-structured CP-nets, and we show that, it can express $(3^n - 2^n)/n$ preference relations by one preference rule on average. Section 5 elaborates the expressive efficiency of equal difference CP-nets, and we prove that its expressive efficiency is 2^{n-1}. Section 6 contains related research aspects from the perspective of graphical model. Finally, in Section 7, we conclude this article and discuss two interesting directions for further theoretical research.

2. Preference representation

Preference handling has become an important topic in many subareas of artificial intelligence [12,5]. In recent years, preference research has received an increasing attention by some periodicals and magazines. For instance, *Computational Intelligence* journal published a special issue on "Preference in Artificial Intelligence (AI) and Constraint Programming (CP)" in issue 2, volume 20 of 2004. *AI* magazine published a special issue on "Preference Handling for Artificial Intelligence" in issue 4, volume 29 of 2008. Besides, *Artificial Intelligence* Journal also published a special issue about "Representing, Processing, and Learning Preferences: Theoretical and Practical Challenges" in issue 7–8, volume 175 of 2011. Very recently, *Machine Learning* journal published a special issue about "Preference Learning and Ranking" in issue 2–3, volume 93 of 2013. From these facts, we can see that researches on preference are very popular.

2.1. Preference representation methods

Preference representation or preference modeling is a prerequisite for preference reasoning [24] and preference aggregation [7]. Along this research direction, we are mainly concerned with how to represent the mathematical structure of preference relations, how to provide an intuitional preferences language for an uninitiated user, and how to get a preferred outcome among some feasible alternatives by a declarative way. By now, there are two types of formalisms to model preference, one is logic model [18,16,19], the other is graphical model [3,6,15] in which the preference elicitation procedure should be as simple as possible. The former always uses the propositional logical language for compactly expressing preference relations over a set of alternatives [20], whereas, the latter uses the intuitive graphical tool for representing dependency or interaction relations among some variables to allow efficient reasoning. The logic method for representing preference is especially suitable for the specification of discriminating assignment satisfying a weighted propositional formula, which uses weight (penalty or reward for violating a goal) to represent the relative importance of propositional formula [18]. Following this spirit, Lang gives the logical preference representation and automated choice of the optimal decision. Additionally, he also illustrates the close relationship between logic preference and combinatorial vote [19]. Coste uses propositional logic languages to represent preference, and studies its expressive power and succinctness [11]. Kaci utilizes a non-monotonic logic to distinguish many kinds of preferences, and uses two kinds of ways—optimistically or pessimistically—to deal with preference uncertainty [16]. Among the popular graphical representation languages, CP-nets [3] are one of the best-known qualitative preference representation languages, all the other graphical representation languages can be regarded as their extensions and derivatives, such as TCP-nets [6] (for tradeoff-enhanced CP-nets, it allows the encoding of conditional relative importance statement between some attributes), CI-nets (representing ordinal preferences over sets of goods [4]), and CP-theory (using $u : x \succ x'[W]$ preference statement to represent "x is preferred to x' irrespective of the values of W when given u and any $t \in V - (X \cup W \cup U)$ " [32]).

2.2. Challenges

Preference representation is a very important research issue in preference handling [12,5], for a specific preference language, it should be emphasized that there are four perspectives to study preference language: complexity, intuition, expressive power and succinctness. As for CP-nets, a mountain of works are concentrated upon exploring the complexity of dominance testing for CP-nets [3,14,30,23], and some fruitful results on reasoning complexity have been obtained [14]. As a graphical model, the intuition of CP-nets is that, vertices represent variables, edges represent dependency relations among variables, and lacking of edges represents conditional preference independence. However, there are few works to discuss the expressive power (or expressivity) and succinctness of CP-nets. As far as we know, investigations on the expressivity and succinctness of CP-nets by a quantitative manner are even rare. Fortunately, studying expressivity and succinctness of some preference proposition languages always occurs in knowledge representation areas, such as Coste and Chevaleyre give the results of expressivity and succinctness on proposition language [11] and weighted proposition language [9] respectively. All these researches encourage and inspire greatly our research interests on expressive efficiency of CP-nets, therefore, based on these works and research line of Lang [19], we study the expressive efficiency of CP-nets. That is, we will give the quantitative trade-off between expressivity and succinctness of CP-nets by a novel concept—expressive efficiency.

2.3. Our approaches and contributions

In this article, we will investigate expressive efficiency of the popular graphical model—CP-nets, which provide qualitative preference information in natural terms. The main contributions of this article can be summarized as follows.