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The Borda Majority Count

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ABSTRACT

In Arrow's framework for social choice the voters are supposed to give a *preference ordering* over the alternatives. In the framework of Michel Balinski and Rida Laraki, called Majority Judgment, as well as in the framework of Warren D. Smith, called Range Voting, voters are supposed to give an *evaluation* of the candidates in some common language or grading system. Consequently, they can convey much more information than in the framework of Arrow. While Warren D. Smith takes for each candidate its average as final value, Balinski and Laraki take the median value for each candidate, in order to reduce the danger of manipulation. However, this brings along a number of (at first sight) counter-intuitive results. As an alternative, we propose in this paper to use a version of the Borda Count, but now in the framework of Balinski and Laraki. We show that the resulting *Borda Majority Count* may be applied for a seat distribution in parliament and has a number of nice properties as well.

1. Introduction

In the traditional framework of Arrow [1–3], where voters are supposed to give a preference ordering over the alternatives, there is no social welfare function that satisfies all of Arrow's properties: Pareto Optimality, Independence of Irrelevant Alternatives (IIA) and non-Dictatorship. In the framework of Arrow, every voter is supposed to give a *ranking* of the alternatives in his own *private* language. For instance, if two voters express the same thing, say that they prefer A to B, they may mean quite different things: one may mean that he has a slight preference for A to B, while the other may mean that he finds A excellent and rejects B.

One of the best, if not the best, voting procedures in the framework of Arrow is the Borda Count [8]: if a voter ranks the alternatives $A_1, \ldots, A_m$ as $A_{\sigma(1)} \succ A_{\sigma(2)} \succ \cdots \succ A_{\sigma(m)}$ (where σ is a permutation of $\{1, \ldots, m\}$), $A_{\sigma(1)}$ gets $m-1$ Borda points, $A_{\sigma(2)}$ gets $m-2$ Borda points, $\ldots, A_{\sigma(m)}$ gets 0 Borda points. The *Borda score* of a given alternative A is the total number of Borda points given by the voters to A. The social ranking of the alternatives and the winner(s) are obtained by comparing the Borda scores of the different alternatives.

In the framework of Balinski and Laraki [4–6], all voters are supposed to give an *evaluation* of each alternative in a *common* language or grading system, understood by everyone in the society. Notice that from an evaluation of the candidates one may deduce a ranking, but conversely, from a ranking of the candidates one cannot deduce an evaluation. So, an evaluation of the candidates is much more informative than a ranking; an evaluation of the candidates enables us to express intensities of preference. However, it is not clear how Balinski and Laraki's Majority Judgment may be used to provide a seat distribution for the different parties in a parliament. Also, although Balinski and Laraki's Majority Judgment (MJ) has many nice

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properties, among others it is independent of irrelevant alternatives in grading (IIAG), it produces in particular cases some results which at first sight may look counterintuitive.

Range Voting, introduced by Smith [11], is similar to Balinski and Laraki's Majority Judgment in that it also assumes a common language. However, in the case of Range Voting the common language consists of a range of scores, while in Majority Judgment the range may also consist of expressions like excellent and good. For instance, the scores may range from 0 till 99, in other words, the common language may be {99, 98, ..., 1, 0}, but smaller ranges are possible, of course. The final score of a candidate or alternative is simply the sum or average of the scores assigned to the candidate by the different voters. The advantage of a large range is, of course, that there is little chance of a tie. On the other hand, the disadvantage of a large range is that it makes manipulation quite easy: if A and B are two close competitors and I have a preference for A, I may give A 99 points and B 0 points, although my sincere grade of B would be 90 points, for instance. Range Voting has many nice properties, as explained in [11], but its weakness is that it is easy to manipulate.

This paper is organized as follows. In Section 2, we explain shortly how the Majority Judgment theory works and discuss two different but closely related tie breaking rules. In Section 3, we propose to apply the Borda Count in the framework of Balinski and Laraki and call this aggregation method the *Borda Majority Count* (BMC). In Section 4 we discuss the properties of the Borda Majority Count and in Section 5 its applications. In Section 6 we discuss some objections to the Borda Majority Count, mainly due to Balinski and Laraki [6]. In the last section, we summarize our results.

2. Majority Judgment

The traditional framework of social choice theory, based on rankings of the alternatives by voters, is riddled with impossibility theorems, saying roughly that a social ranking function or choice function with only nice properties cannot exist [1,2]. Balinski and Laraki [6] on the one hand and Smith [11] on the other hand proposed a new framework in which voters are not asked to give their preferences over the alternatives, but instead they are asked to give *evaluations* of all candidates in a *common* language or grading system understood by everyone in society. This is what happens in many contests in real life. Notice that in this way the information provided by the voters is much more informative than in the traditional framework of Arrow. Two voters who both prefer A to B may express their opinion in more detail: one may judge that A is excellent and B is very good, while the other may judge that A is very good and B is very poor. The difference between Balinski and Laraki's *Majority Judgment* and Smith' *Range Voting* is that in the first case the median or middlemost value, and in the second case the average, of the grades given to a candidate is taken as the final grade of that alternative.

Below we present Majority Judgment in a compact form. Since the occurrence of ties is quite likely, tie breaking rules are needed. Balinski and Laraki present two different, although closely related, tie breaking rules.

2.1. The framework of Majority Judgment

As in the traditional framework, we assume a finite set $\mathcal{C}$ of m competitors, candidates or alternatives, $A_1, A_2, \ldots, A_m$, and a finite set J of n judges or voters, $1, 2, \ldots, n$. Furthermore, a *common language* $\mathcal{L}$ is a finite set of strictly ordered grades g_i, or an interval of the real numbers. We take $g_i \geqslant g_j$ to mean that g_i is a higher grade than g_j or $g_i = g_j$.

The input for a *method of grading* is an m by n matrix, called a *profile*, filled with grades g_{ij} in $\mathcal{L}$, where g_{ij} denotes the grade that judge j assigns to alternative A_i. So, row i in the profile contains the grades given by the different judges to alternative A_i, while column j contains the grades that judge j gives to the different alternatives. A *method of grading* is a function F that assigns to every profile the final grade of every competitor.

More precisely, let $f : \mathcal{L}^n \to \mathcal{L}$, then the *method of grading* (determined by f) is the function $F : \mathcal{L}^{m\times n} \to \mathcal{L}^m$ such that

$$\begin{pmatrix} g_{11} & g_{12} & \cdots & g_{1n} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ g_{m1} & g_{m2} & \cdots & g_{mn} \end{pmatrix} \to^F (f(g_{11}, g_{12}, \ldots, g_{1n}), \ldots, f(g_{m1}, g_{m2}, \ldots, g_{mn}))$$

where $f(g_{i1}, g_{i2}, \ldots, g_{in})$ is called the final grade of competitor A_i and f is called an *aggregation function.*

The **majority grade** $f^{maj}(A)$ of a candidate A is defined as follows:

If $g_1 \geqslant \cdots \geqslant g_n$ are the grades given to candidate A and n is odd, then $f^{maj}(A) = g_{(n+1)/2}$, i.e., the middlemost grade: there is a majority that judges the grade should be at least $g_{(n+1)/2}$ and there is another majority that judges the grade should be at most $g_{(n+1)/2}$. So, if A gets grades 9, 8, 6, 4, 3, then $f^{maj}(A) = 6$. Notice that by taking the middlemost value, Majority Judgment resists manipulation: the judge that gave A a 3 and thinks that a 6 is too high cannot lower A's majority grade by giving a grade lower than 3; similarly, the judge that gave A an 8 and thinks that a 6 is too low, cannot raise A's majority grade.

If n is even, then the middlemost interval is defined as $[g_{(n+2)/2}, g_{n/2}]$, where $g_{n/2}$ is the upper middle most grade and $g_{(n+2)/2}$ is the lower middlemost grade. So, if A receives the grades 9, 7, 6, 4, 3, 2 from six judges, the middlemost interval is [4,6]. Every grade other than a grade in the middle-most interval is condemned by an absolute majority of the judges as being either too high or too low. In the case of an even number of voters, Balinski and Laraki [6] argue that the majority grade $f^{maj}(A)$ of an alternative A should be defined as the lower middlemost grade $g_{(n+2)/2}$. So, for instance, when the grades assigned to alternative A are 9, 7, 6, 4, 3, 2, then $f^{maj}(A) = 4$.