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On bisimulations for description logics ☆

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ABSTRACT

We study bisimulations for useful description logics. The simplest among the considered logics is $\mathcal{ALC}_{reg}$ (a variant of PDL). The others extend that logic with the features: inverse roles, nominals, qualified number restrictions, the universal role, and/or the concept constructor for expressing the local reflexivity of a role. They also allow role axioms. Our contributions are as follows. We propose to treat named individuals as initial states and give an appropriate bisimulation condition for that. We also give bisimulation conditions for the universal role and the concept constructor $\exists r.\mathtt{Self}$. We prove that all of the bisimulation conditions for the features can be combined together to guarantee invariance of concepts and the Hennessy-Milner property for the whole class of studied description logics. We address and give results on invariance or preservation of ABoxes, RBoxes and knowledge bases in description logics. Independently from Lutz et al. [26,27] we also give results on invariance of TBoxes. We introduce a new notion called QS-interpretation, which is needed for dealing with minimizing interpretations in description logics with qualified number restrictions and/or the concept constructor $\exists r.\mathtt{Self}$. We formulate and prove results on minimality of quotient interpretations w.r.t. the largest auto-bisimulations. We adapt Hopcroft's automaton minimization algorithm to give an efficient algorithm for computing the partition corresponding to the largest auto-bisimulation of a finite interpretation in any description logic of the considered family. Using the invariance results we compare the expressiveness of the considered description logics w.r.t. concepts, TBoxes and ABoxes. Our results about separating the expressiveness of description logics are naturally extended to the case when instead of $\mathcal{ALC}_{reg}$ we have any sublogic of $\mathcal{ALC}_{reg}$ that extends $\mathcal{ALC}$.

1. Introduction

Description logics (DLs) are variants of modal logic. They are of particular importance in providing a logical formalism for ontologies and the Semantic Web. DLs represent the domain of interest in terms of concepts, individuals, and roles. A concept is interpreted as a set of individuals, while a role is interpreted as a binary relation among individuals. A DL is characterized by a set of concept constructors, a set of role constructors, and a set of allowed forms of role axioms and individual assertions. A knowledge base in a DL usually has three parts: an RBox consisting of axioms about roles, a TBox consisting of terminology axioms, and an ABox consisting of assertions about individuals. The basic DL $\mathcal{ALC}$ allows basic concept

☆ This is an extended version of the workshop paper [14].

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Table 1
Concept constructors for $\mathcal{ALC}$ and some additional constructors/features of other DLs.

Constructor	Syntax	Example
Concept constructors of $\mathcal{ALC}$		
complement	$\neg C$	$\neg Male$
intersection	$C \sqcap D$	$Human \sqcap Male$
union	$C \sqcup D$	$Doctor \sqcup Lawyer$
existential restriction	$\exists r.C$	$\exists hasChild.Male$
universal restriction	$\forall r.C$	$\forall hasChild.Female$
Constructor/Feature	Syntax	Example
Some additional constructors/features of other DLs		
inverse roles ($\mathcal{I}$)	r^-	$hasChild^-$ (i.e., $hasParent$)
qualified number	$\geq n\ R.C$	$\geq 3\ hasChild.Male$
restrictions ($\mathcal{Q}$)	$\leq n\ R.C$	$\leq 2\ hasParent.\top$
nominals ($\mathcal{O}$)	$\{a\}$	$\{John\}$
hierarchies of roles ($\mathcal{H}$)	$R \sqsubseteq S$	$hasChild \sqsubseteq hasDesc$
transitive roles ($\mathcal{S}$)	$R \circ R \sqsubseteq R$	$hasDesc \circ hasDesc \sqsubseteq hasDesc$

constructors listed in Table 1, but does not allow role constructors nor role axioms. The most common additional features for extending $\mathcal{ALC}$ are also listed in Table 1.

Given two individuals in an interpretation, sometimes we are interested in the question whether they are "similar" or not, i.e., whether they are indiscernible w.r.t. the considered description language. Indiscernibility is used, for example, in machine learning. In DLs, it is formally characterized by bisimulation. Roughly speaking, two individuals are indiscernible iff they are bisimilar.

Bisimulations arose in modal logic [37–39] and state transition systems [19,30]. They were introduced by van Benthem under the name *p-relation* in [37,38] and the name *zigzag relation* in [39]. Bisimulations reflect, in a particularly simple and direct way, the locality of the modal satisfaction definition. The famous Van Benthem Characterization Theorem states that modal logic is the bisimulation invariant fragment of first-order logic. Bisimulations have been used to analyze the expressivity of a wide range of extended modal logics (see, e.g., [4] for details). In state transition systems, bisimulation is viewed as a binary relation associating systems which behave in the same way in the sense that one system simulates the other and vice versa. Kripke models in modal logic are a special case of labeled state transition systems. Hennessy and Milner [19] showed that weak modal languages could be used to classify various notions of process invariance. In general, bisimulations are a very natural notion of equivalence for both mathematical and computational investigations.[1]

Bisimilarity between two states is usually defined by three conditions (the states have the same label, each transition from one of the states can be simulated by a similar transition from the other, and vice versa). As shown in [4], the four program constructors of PDL (propositional dynamic logic) are "safe" for these three conditions. That is, we need to specify the mentioned conditions only for atomic programs, and as a consequence, they hold also for complex programs. For bisimulation between two pointed-models, the initial states of the models are also required to be bisimilar. When converse is allowed (the case of CPDL), two additional conditions are required for bisimulation [4]. Bisimulation conditions for dealing with graded modalities were studied in [10,11,22]. In the field of hybrid logic, the bisimulation condition for dealing with nominals is well known (see, e.g., [1]).

In this paper we study bisimulations for the family of DLs which extend $\mathcal{ALC}_{reg}$ (a variant of PDL) with an arbitrary combination of inverse roles, qualified number restrictions, nominals, the universal role, and the concept constructor $\exists r.\mathtt{Self}$ for expressing the local reflexivity of a role. Inverse roles are like converse modal operators, qualified number restrictions are like graded modalities, and nominals are as in hybrid logic.

The topic is worth studying due to the following reasons:

1. Despite that bisimulation conditions are known for PDL and for some features like converse modal operators, graded modal operators and nominals, we are not aware of previous work on bisimulation conditions for the universal role and the concept constructor $\exists r.\mathtt{Self}$. More importantly, without proofs one cannot be sure that all the conditions can be combined together to guarantee standard properties like invariance and the Hennessy-Milner property.
   There are many papers on bisimulations, but just a few on bisimulations in DLs:
   - In [23] Kurtonina and de Rijke studied expressiveness of concept expressions in some DLs by using bisimulations. They considered a family of DLs that are sublogics of the DL $\mathcal{ALCNR}$, which extend $\mathcal{ALC}$ with (unqualified) number restrictions and role conjunction. They did not consider individuals, nominals, qualified number restrictions, the concept constructor $\exists r.\mathtt{Self}$, the universal role, and the role constructors like the program constructors of PDL.

[1] This paragraph is based on [4].