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Interpretation of equilibria in game-theoretic rough sets

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ABSTRACT

Intelligent decision making models aim at improving the quality of decision making under uncertainty. The fundamental issues that are generally encountered in these models are too many options to choose from and the involvement of contradictory decision making criteria. The game-theoretic rough set (GTRS) model provides an intelligent decision making mechanism that exploits a game-theoretic environment for analyzing strategic situations between cooperative or conflicting decision making criteria in the probabilistic rough set framework. The concept of equilibria is of central importance in the GTRS model which has not been sufficiently addressed in the current literature. Two key issues in this regard are the interpretation of equilibria and the establishment of their existence. By reviewing, examining and defining the basic game constructs in the GTRS model, we are able to interpret an equilibrium in terms of the decision thresholds that control the rough sets based decision regions. In particular, an equilibrium is defined in terms of a pair of thresholds such that no player has a unilateral incentive to change these thresholds within the game. An example game is considered to demonstrate the use of the interpretation in determining the thresholds. The issue of existence of equilibria is addressed by considering a couple of typical two-player games in the GTRS model. The results suggest that the existence of equilibria may be established under certain limited conditions.

1. Introduction

Uncertainty is a critical factor that affects decision making and real world problem solving [22]. It may occur due to incomplete, imprecise or contradictory information, varying test results, approximate measurements or partially observable environment, just to mention some causes. Intelligent decision making is an interdisciplinary research where one of the primary concerns is to develop models for assisting in effective decision making [21,24]. Two commonly encountered issues in these models are the presence of too many yet potentially useful choices or alternatives and the involvement of conflicting or contradictory criteria for evaluating possible decisions [19,25]. The game-theoretic rough set (GTRS) model is a recent alternative for intelligent decision making that meet these challenges and issues by taking advantage from rough sets based data analysis coupled with game-theoretic decision analysis [17,18,23,28].

An important contribution of GTRS is that it provides a mechanism for determining a pair of thresholds that are used in the probabilistic rough set model to induce three-way decisions [8]. Given a certain hypothesis and a pair of thresholds (α, β), if the probability of a hypothesis is above an upper threshold we accept the hypothesis; if it is below a lower threshold we reject the hypothesis; if it is in between the two thresholds we choose a third decision called non-commitment, deferment or

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uncertain decision [9,14]. One can expect more deferred decisions in the presence of uncertain information. The pair of thresholds provides a tradeoff between certain versus deferred decisions based on the quality of available information. A fundamental problem in this context is the interpretation and determination of probabilistic thresholds [33]. Some notable attempts in this regard include decision-theoretic, game-theoretic, information-theoretic, optimization based and risk based approaches [3,5,10,13,15,27,30].

It is worth mentioning that Yao and Deng recently presented a framework of quantitative rough sets based on subsethood or set inclusion measures [34]. The framework provides a platform to unify the study of some of the existing generalizations of the rough set models. Despite having different formulation of the quantitative rough sets, the rough set approximations and the three regions are defined by a pair of thresholds on subsethood measures. The determination of thresholds remains as a challenge in the new framework.

The GTRS based thresholds determination provides benefits in at least two aspects. Firstly, the ability to determine the threshold levels based on a tradeoff solution between multiple criteria can comparatively lead to more effective and moderate threshold levels [2,7,26,35]. Secondly, unlike other models which rely on user-provided parameters (such as the variable precision rough set model [11,36]) or utilize the notions such as the costs, risks or uncertainty to determine the thresholds (such as the decision-theoretic rough set model and its extensions [16,20,29,30]), the GTRS learns the thresholds based on the data itself [7].

The equilibria play a critical role in determining thresholds in the GTRS model. Although the existing studies demonstrate the usefulness of GTRS in determining the thresholds, there is still a gap in relating the possible game solutions with the computed or determined thresholds. This leads to the issue of interpretation of equilibria in the GTRS model. The game solutions are used in GTRS to determine the thresholds. Although it was possible with existing games in GTRS to determine the thresholds based on the game solutions, however, it may not be clear whether a game solution always exists. In other words, we are not able to rule out the possibility of having no game solution. This leads to the issue of establishment of equilibria in the GTRS model. In this article, we focus on these two issues.

To address the meaning of equilibria, we first defined and provided interpretation of the basic game components including the players, strategies and strategy profiles in GTRS. The strategies are defined as representing possible changes or modifications in the thresholds and the strategy profiles as different threshold pairs representing various probabilistic rough set models. Based on these descriptions, an equilibrium is interpreted as a strategy profile such that for the corresponding thresholds, no player has any incentive or gain to change these thresholds within the game. The issue of the establishment of equilibria is addressed by considering a couple of typical two-player games in GTRS. It is suggested that the existence of equilibria may be established under certain limited conditions.

The implications and insights gained from the results in this study will help in resolving some semantic issues with GTRS. As a result, we expect advantages that are twofold: firstly, it will help in improving the current understandability of the model and secondly, it will lead to a better awareness about the theory, its limitations and appropriateness to different real life problems. The remaining of this paper is organized as follows. Section 2 presents some of the background knowledge about the GTRS and highlights the role of equilibria in the model. Section 3 is on issue of interpretation of a game solution. Section 4 is on the issue of establishment of equilibria.

2. Equilibria in game-theoretic rough set model

The probabilistic rough sets define the lower and upper approximations for a concept C using a pair of thresholds (α, β) [31,33],

$$\underline{apr}_{(\alpha,\beta)}(C) = \{x \in U \mid P(C|[x]) \geqslant \alpha\}, \tag{1}$$

$$\overline{apr}_{(\alpha,\beta)}(C) = \{x \in U \mid P(C|[x]) > \beta\}, \tag{2}$$

where U is the set of objects called universe and $E \subseteq U \times U$ is an equivalence relation on U. The equivalence class of E containing object $x \in U$ is denoted as $[x]$ and the concept $C \subseteq U$. The three rough set regions based on lower and upper approximations are defined as,

$$\mathrm{POS}_{(\alpha,\beta)}(C) = \underline{apr}_{(\alpha,\beta)}(C) = \{x \in U|P(C|[x]) \geqslant \alpha\}, \tag{3}$$

$$\mathrm{NEG}_{(\alpha,\beta)}(C) = (\overline{apr}_{(\alpha,\beta)}(C))^c = \{x \in U|P(C|[x]) \leqslant \beta\}, \tag{4}$$

$$\mathrm{BND}_{(\alpha,\beta)}(C) = \overline{apr}_{(\alpha,\beta)}(C) - \underline{apr}_{(\alpha,\beta)}(C) = \{x \in U|\beta < P(C|[x]) < \alpha\}, \tag{5}$$

where $P(C|[x])$ denotes the conditional probability of an object x to be in C given that the object is in $[x]$ and $0 \leqslant \beta < \alpha \leqslant 1$. In contrast to the conventional Pawlak model, where $x \in C$ is true for the positive region, false for the negative region and neither true nor false for the boundary region, the probabilistic approach utilizes a quantitative characteristic of an object belonging to a region in the form of probabilistic association or membership level. Particularly, an object x is considered to be in C if its probabilistic association with C is at or above the level α, i.e., $P(C|[x]) \geqslant \alpha$. The same object is considered not to be in C if its probabilistic association is at or below the level β, i.e., $P(C|[x]) \leqslant \beta$. With the current information whether or not the object x to be in C can not be determined when the probabilistic association of x with C is between the two thresholds, i.e., $\beta < P(C|[x]) < \alpha$.