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On the relation between rough set reducts and typical testors

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ABSTRACT

This paper studies the relations between rough set reducts and typical testors from the so-called logical combinatorial approach to pattern recognition. Definitions, comments and observations are formally introduced and supported by illustrative examples. Furthermore, some theorems expressing theoretical relations between reducts and typical testors are enunciated and proved. We also discuss several practical applications of these relations that can mutually enrich the development of research and applications in both areas. Although we focus on the relation between the classical concepts of testor and reduct, our study can be expanded to include other types of testors and reducts.

1. Introduction

Object classification is usually preceded by feature selection, which reduces the dimension of the space of representation of objects. Intensive research on feature selection and pattern discovery has given us several new representations and approaches that use different formalisms. We have decided to examine the relations between two concepts from the domain of knowledge discovery: reducts and typical testors. Typical testors derive from the Test Theory [6,64] and the logical combinatorial approach to pattern recognition [46] whilst reducts refer to rough set theory [39,40]. Both concepts are still being expanded in various directions [30,41–43,56].

The assumption that both concepts are closely related is based on their common properties. In both approaches, *decision tables* are used for data representation, it means that objects of study are perceived by means of information represented by attributes. Both, reducts and typical testors, represent strong differentiating power, they are attribute subsets jointly sufficient and individually necessary to discern among object descriptions.

When considering decision tables, it is important to distinguish between the so called *consistent* and the *inconsistent* ones. A decision table is said to be *consistent*, if each combination of values of descriptive attributes uniquely determines the value of the decision attribute, and *inconsistent*, otherwise.

As an important antecedent of our work, we can mention the work of Moshkov et al., see for example [7,33,34], who have deeply studied decision trees and relations among testors, decision rules and decision trees. Nevertheless, in these publications, authors only take into account consistent tables while we consider both consistent and inconsistent ones.

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We study the relation between reducts and testors and enunciate several practical applications that can be enriched by developing research and applications in both areas. Although we focus on the relation between the classical concepts of testor and reduct, our study can be expanded to include other types of testors and reducts.

Most definitions and properties presented in this paper are not new. The novelty of this study is to put all these definitions into a unified framework; emphasizing the coupling issues between reduct and testor definitions. Through our study, the relations among reduct definitions and testor definitions are more easily recognized. This view can enhance the theoretical and logical understanding of both basic concepts. As a result, algorithms and software developed to calculate either typical testors or reducts can be used interchangeably for both tasks, considering the specifications and characteristics studied here.

This document is organized as follows. Section 2 provides the formal background for the study of reducts and testors within the rough set theory and the logical combinatorial approach to pattern recognition, respectively. In this section, we include several examples for supporting our discussion. Section 3 contains a list of theorems expressing relations between reducts and typical testors, including a case study. In Section 4, we discuss practical applications of the presented relations and future research directions. Our remarks are summarized in Section 5.

2. Theoretical foundations

This paper examines relations between reducts and typical testors related to the same classification data. Since both concepts originated from separate fields of knowledge discovery, they are usually expressed in different formalisms. Reducts are a key concept within the rough set theory, defined for information systems or decision tables. On the contrary, typical testors arose as a tool to detect faults in contact outlines and later their applicability transcended to be used in the solution of feature selection and classification problems. The purpose of this section is to provide a formal view on the ideas according to their original fields studying the equivalence between both formalisms.

2.1. Reducts

In many data analysis applications, information and knowledge are stored and represented as an information table because information tables provide a convenient way to describe a finite set of objects within a universe through a finite set of attributes [39].

Definition 1 (*Information system*). An information system is a pair $\mathcal{S} = (U, A_t)$ where U is a finite non-empty set of objects, A_t is a finite non-empty set of attributes. Each $a \in A_t$ corresponds to the function $I_a : U \to V_a$ called evaluation function, where V_a is called the value set of a.

An *information table* is the simplest form of an information system. It can be implemented as a two-dimensional array (matrix), which is a standard data structure in any programming language. In an information table, we usually associate its rows to objects, its columns to attributes and its cells to values of attributes on objects.

Definition 2 (*Indiscernibility relation*). Given a subset of attributes $A \subseteq A_t$, the indiscernibility relation $IND(A) \subseteq U \times U$ is defined as

$$IND(A) = \{(u, v) \in U \times U : \forall\ a \in A; I_a(u) = I_a(v)\}$$

For any two objects $u, v \in U$, if $(u, v) \in IND(A)$, then u and v are indiscernible based on the attribute set A. The indiscernibility relation is an equivalence relation, so it induces a partition $U/IND(A)$ over U.

According to the indiscernibility relation, Pawlak defined a reduct in an information system as a minimal set of attributes that keeps the indiscernibility relation $IND(A_t)$ unchanged [40].

Definition 3 (*Reduct for an information system*). Given an information system $\mathcal{S} = (U, A_t)$, an attribute set $R \subseteq A_t$ is called a reduct if R satisfies the following two conditions:

(i) $IND(R) = IND(A_t)$;
(ii) For any $a \in R, IND(R - \{a\}) \neq IND(A_t)$.

If R satisfies (i) it is called a super reduct.

The first condition indicates the joint sufficiency of the attribute set R. The second condition indicates that each attribute in R is individually necessary.

Definition 4 (*Decision table*). A special type of information table is denoted as $\mathcal{S}_d = \left(U, A_t = A_t^* \cup \{d\}\right)$, where A_t^* is a set of conditional attributes and d is a decision attribute indicating the decision class for each object in the universe ($\{d\} = D$). Such information table is called a decision table.