Information Sciences 294 (2015) 305–314

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Sequential aggregation of bags

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ARTICLE INFO

Article history:
Received 12 March 2013
Received in revised form 16 May 2014
Accepted 17 September 2014
Available online 7 October 2014

Keywords:
Aggregation function
Bag
Participatory learning
Sequential aggregation

ABSTRACT

This study introduces and develops a new approach to sequential aggregation of bags. It generalizes several related approaches known from the literature. The approach is based on a system of aggregation functions with property (SA) – a weakened form of symmetry. It generalizes symmetric associative aggregation. In the paper, several methods of constructing sequential aggregations, as well as extended aggregation functions with property (SA), are introduced and exemplified. Moreover, an example of a real-world application, which illustrates the proposed sequential aggregation procedure, is included.

1. Introduction

Processing of data that come in some groups one-by-one requires special approaches to aggregation. That occurs in many real-world problems where data arrive in packages, which come from the same time, place or relative position. For example, time observation suggests data that come perhaps day by day; a geographical observation suggests data that come perhaps location by location; and image processing might suggest that each particular pixel should be understood from the information contained in surrounding pixels (i.e., in packages which depend on the distance to such pixel, as pointed in [25]). A related problem is processing of data which can be classified into several disjoint classes with hierarchical structure. In this paper, we will deal with aggregation of groups of data $\mathbf{x}_1, \ldots, \mathbf{x}_m \in \bigcup_{n\in\mathbb{N}}[0,1]^n$ which are obtained one-by-one. The ith group of data is denoted by $\mathbf{x}_i = (x_{i1}, \ldots, x_{in_i})$. The data in a group can be repeated and their ordering is not important, each piece of information in the given group has the same importance. Such groups are called bags. In aggregation theory, the idea of bags was firstly considered by Yager [31]. Formally, bags can be viewed as multisets, i.e., sets of the form $\{(v_1, m_1), \ldots, (v_k, m_k)\}$, where $v_1, \ldots, v_k$ are different input arguments and $m_1, \ldots, m_k$ are their respective occurrences ($m_i \in \{1, 2, \ldots\}$). As in the case of bags the order of arguments does not matter, they can be represented as m-tuples $(x_1, \ldots, x_m)$, where $m = m_1 + m_2 + \ldots + m_k$ and $\mathrm{card}\{j | x_j = v_i\} = m_i, j \in \{1, \ldots, m\}, i \in \{1, \ldots, k\}$. Clearly, for any permutation $\sigma : \{1, \ldots, m\} \to \{1, \ldots, m\}$, m-tuples $(x_1, \ldots, x_m)$ and $(x_{\sigma(1)}, \ldots, x_{\sigma(m)})$ represent the same bag. In this paper, we do not distinguish bags $\{(v_1, m_1), \ldots, (v_k, m_k)\}$ and their representations $(x_1, \ldots, x_m)$.

Consider that in the beginning we only have at disposal the data contained in the first bag $\mathbf{x}_1$ (e.g., "the first day observations"), and then the information is completed by a bag of data $\mathbf{x}_2 = (x_{21}, \ldots, x_{2n_2})$ ("the second day observations"), etc. Our

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http://dx.doi.org/10.1016/j.ins.2014.09.046
0020-0255/© 2014 Published by Elsevier Inc.

task is to gain a most informative value $a_m \in [0,1]$, that characterizes the considered data $\mathbf{x}_1,\ldots,\mathbf{x}_m$ so that if we obtain a new bag of data $\mathbf{x}_{m+1} \in [0,1]^{n_{m+1}}$, we can determine the value a_{m+1} from a_m and $\mathbf{x}_{m+1}$. Note that similar problems are discussed in learning problems in neural systems, see, e.g., [18,33–35]. To capture a special nature of aggregation of data that come in bags, one can use aggregation functions which have some grouping properties, such as associativity, decomposability or bisymmetry, for more details see [2,3,13–15]. More special approaches based on composite aggregation or on recursive formulas can be found in [1,5,7–9,11,34]. A rather general approach – called sequential aggregation – was recently proposed by Kolesárová in [17]. Some related ideas can also be found in [19,27].

The aim of this paper is a substantial development and generalization of the original idea of sequential aggregation, including proposals of several easy to be implemented sequential aggregation procedures. The paper is organized as follows: In the next section, the concept of sequential aggregation is introduced, including an illustrative example. Section 3 is devoted to some construction methods of sequential aggregations based on the averaging or conjunctive approaches. Several constructions of special aggregation functions that satisfy the (SA) property introduced in Section 2 are described in Section 4. Finally, some concluding remarks are provided.

2. Sequential aggregation

Aggregation of a finite set of input values from the unit interval $[0,1]$ is a substantial tool in many theoretical and applied areas. We restrict our theoretical considerations to the unit interval, but one could also consider a general interval $I \subseteq [-\infty,\infty]$. In this contribution we follow the terminology used in the recent monograph [13], which describes state-of-the-art of aggregation theory in the last decade.

Definition 1. An extended aggregation function B is a mapping $B : \bigcup_{n\in\mathbb{N}}[0,1]^n \to [0,1]$ that satisfies the properties:

(i) For each $x \in [0,1], B(x) = x$,
(ii) For each number $n \in \mathbb{N}$ of inputs, $B(\underbrace{0,\ldots,0}_{n\text{–}times}) = 0, \quad B(\underbrace{1,\ldots,1}_{n\text{–}times}) = 1$,
(iii) For each $n \in \mathbb{N}, B(x_1,\ldots,x_n) \leqslant B(y_1,\ldots,y_n)$ whenever $(x_1,\ldots,x_n), (y_1,\ldots,y_n) \in [0,1]^n$, such that $x_i \leqslant y_i$, for each $i = 1,\ldots,n$.

For a fixed $n \in \mathbb{N}, n \geqslant 2$, a $[0,1]^n \to [0,1]$ mapping, which satisfies the boundary conditions (ii) and increasing monotonicity (iii) mentioned in the previous definition, is called an n-ary aggregation function. More information on aggregation functions can be found in [13], see also [2,3]. A discussion on standard assumptions can be found in [12].

Let us suppose that the first bag $\mathbf{x}_1$ contains n_1 data, $\mathbf{x}_1 = (x_{11}, x_{12},\ldots,x_{1n_1}) \in [0,1]^{n_1}$, and let the result of their aggregation be the value $a_1 = B_1(\mathbf{x}_1)$. In the next step, we aggregate the value a_1 and the data of the second bag $\mathbf{x}_2 = (x_{21}, x_{22},\ldots,x_{2n_2}) \in [0,1]^{n_2}$ using an extended aggregation function B_2. We obtain the value

$$a_2 = B_2(a_1,\mathbf{x}_2) = B_2(a_1, x_{21}, x_{22},\ldots,x_{2n_2}).$$

We proceed in this manner in each step, thus the result of the ith aggregation, $i \geqslant 2$, is

$$a_i = B_i(a_{i-1},\mathbf{x}_i) = B_i(a_{i-1}, x_{i1}, x_{i2},\ldots,x_{in_i}).$$

As mentioned above, all elements $x_{i1},\ldots,x_{in_i}$ of the ith bag of information (for each i) are equivalent with respect to the order, therefore the aggregation function B_1 must be symmetric and all $B_i, i = 2,3,\ldots$, must satisfy the property

$$B_i(a_{i-1}, x_{i1},\ldots,x_{in_i}) = B_i(a_{i-1}, x_{i\sigma(1)},\ldots,x_{i\sigma(n_i)})$$

for any permutation σ of indices $(1,\ldots,n_i)$.

Definition 2. Let B be an extended aggregation function. We say that B has property (SA), if for all $(u_1,\ldots,u_{n+1}) \in [0,1]^{n+1}$ and each permutation σ of the n-tuple $(2,\ldots,n+1)$ it holds that

$$B(u_1,u_2,\ldots,u_{n+1}) = B(u_1,u_{\sigma(2)},\ldots,u_{\sigma(n+1)}).$$

Clearly, each symmetric extended aggregation function B has property (SA), i.e., (SA) is a weakened form of symmetry. The property (SA) is crucial for our idea of sequential aggregation which indicates the origin of its notation (Sequential Aggregation).

Example 1. The aggregation function B, which is given by

$$B(u_1,\ldots,u_{n+1}) = u_1 \min\{u_2,\ldots,u_{n+1}\},$$

is an aggregation function with property (SA). Note that this extended aggregation function is not symmetric.

This example suggests a more general situation that assures the property (SA) of extended aggregation functions, namely the case when the aggregation function B can be decomposed in terms of two functions,