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Incremental update of approximations in dominance-based rough sets approach under the variation of attribute values

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ABSTRACT

Dominance-based Rough Sets Approach (DRSA) has received much attention since it is able to acquire knowledge from information with preference ordered attribute domains and decision classes. In many real-life applications, the information systems may evolve over time dynamically. In a dynamic information system, the obtained knowledge, e.g., approximations in DRSA, need to be updated for decision making and other related tasks. As a useful technique, the incremental update can be applied to process dynamic information with revising the obtained knowledge. In this paper, we propose an incremental approach for maintaining approximations of DRSA when attribute values vary over time. Some numerical examples illustrate that the incremental approach can renew approximations of DRSA without beginning from scratch. Experimental evaluations show that the incremental algorithm can effectively reduce the computational time in comparison with the non-incremental one when the ratio of the attribute values varied is less than a threshold.

1. Introduction

Rough Sets Theory (RST) introduced by Pawlak in the early 1980s can be used to process inconsistent information [36,37]. The key of RST is an indiscernibility relation, which partitions the object set of an information system into a collection of equivalence classes. By RST, an equivalence class is the smallest information unit of the information system, called the information granularity. The objects that belong to an equivalence class are indiscernible with respect to the available information. If a set can be exactly described by equivalence classes, then it is a crisp set; otherwise, it is a rough set. Any rough set can be characterized by two crisp sets: its lower and upper approximations. For any rough set X, its lower approximation is a subset of X, and its upper approximation contains X.

Many scholars generalized RST to handle various kinds of real-life problems [38]. Yao presented Decision Theoretic Rough Set (DTRS) based on the well-established Bayesian decision procedure [52]. It is a general probabilistic rough set model since different probabilistic rough set models may be derived from DTRS [51]. Ziarko proposed Variation Precision Rough Sets (VPRS) model in order to smooth away the influence of imprecise data to knowledge acquisition [60]. To deal with incomplete data, three extensions of RST were introduced as follows: an early extension of rough sets that can directly deal with incomplete data presented by Kryszkiewicz in [24] is under a tolerance relation. Stefanowski et al. extended the model of rough sets by using of a non-symmetric similarity relation in [47]. Grzymala-Busse proposed an extension of rough sets in terms of a

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characteristic relation under the assumption that some of the missing attribute values are lost (*e.g.*, they were erased) and some are "do not care" conditions (*e.g.*, they are redundant or unnecessary to make a decision or to classify a case) [19]. Dubois and Prade generalized the concept of rough sets to the fuzzy environment and initiated the concepts of rough fuzzy sets and fuzzy rough sets [12]. Greco et al. proposed Dominance-based Rough Sets Approach (DRSA) to deal with missing data in multi-attribute and multi-criteria decision problems [14,15] and process information systems with preference-ordered attribute domains and decision classes [16]. Qian et al. extended Pawlak's rough set model to a multi-granulation rough set model (MGRS) [43]. Following Qian's viewpoint, Lin et al. extended MGRS from partition to covering [28].

As a kind of method that can be applied to assist multiple criteria decision making, DRSA has been received much attention. For example, Kotlowski et al. found that the notions of rough approximations of DRSA are excessively restrictive for the presence of noise in real-life problems. They proposed a probabilistic model based on DRSA to the problem of ordinal classification with monotonicity constraints [23]. Dembczynski et al. considered imprecise evaluations of objects on condition criteria and imprecise assignments of objects to decision classes. They reformulated the dominance principle and proposed second-order rough approximations [11]. Yang et al. proposed a novel dominance relation in incomplete interval-valued information systems [50]. For interval-valued information systems, Qian et al. introduced another ordering approach named as Kulisch-Miranker order [41], and Huang et al. proposed a graded dominance interval-valued relation based rough sets [21].

In some real-life problems, the information systems evolve with time since data is continually collected. For a dynamic information system, the granules related to knowledge acquirement may evolve over time [39]. In order to apply RST in dynamic information processing, many researchers integrated the incremental update technique to RST or its extensions [3–5,7–10,18,20,22,25–27,32,33,35,45,49,54,55,59]. Their studies may be divided into three main aspects as follows:

- Variation of the object set
  The incremental approaches for updating knowledge under the variation of the object set were extensively studied in recent years. Some excellent incremental approaches based on Pawlak's rough sets model were reported in [3,20,45,49,59]. In addition, some noticeable incremental approaches were proposed for extensions of Pawlak's rough sets model as follows. Liu et al. presented a VPRS-based incremental model and approach as well as its algorithm for inducing interesting knowledge when the object set varies over time. Furthermore, in business intelligent information systems, Liu et al. proposed a VPRS-based optimization incremental approach as well as its algorithm for inducing interesting knowledge [32,33]. Chen et al. proposed a new incremental method for updating approximations of VPRS while objects in the information system dynamically alter [9]. Jerzy et al. discussed the incremental induction of decision rules from dominance-based rough approximations to select the most interesting representatives in the final set of rules [4]. Greco et al. investigated an incremental induction of decision rules in the context of multiple criteria decision analysis [18]. Jia et al. proposed an incremental algorithm INRIDDM based on DRSA. When a new object arrives, the updated rule sets could be obtained after updating one row or column in a multi-dominance discernibility matrix [22]. Li et al. proposed an incremental approach for updating approximations of DRSA under variation of the object set [25]. Zhang et al. presented a new dynamic method for incrementally updating approximations of a concept under neighborhood rough sets to deal with numerical data [54]. Luo et al. proposed an incremental method for updating approximations in set-valued ordered information systems when the information system is updated by inserting or deleting objects [35]. Zhang et al. developed a novel matrix-based algorithm for fast updating approximations in dynamic composite information systems [53].
- Variation of the attribute set
  Chan firstly proposed an incremental algorithm for learning classification rules efficiently when an attribute set in the information system evolves over time [5]. Under a characteristic relation, Li et al. proposed an incremental approach for updating approximations of a concept by considering adding or removing some attributes simultaneously in the incomplete information systems [27]. Qian et al. introduced a theoretic framework based on rough set theory, called positive approximation, which can be used to accelerate a heuristic process of attribute reduction [42]. Cheng proposed two incremental approaches for updating approximations in rough fuzzy sets [10]. Zhang et al. presented a rough sets based matrix approach for handling the dynamic attribute set in the set-valued information system [55]. Li et al. proposed an incremental approach for updating approximations of DRSA on the variation of attributes [26]. Luo et al. presented an approach for maintaining approximations dynamically in set-valued ordered decision systems under the attribute generalization [34].
- Refining or coarsening of the attribute values
  The refining or coarsening as a special case on the variation of the attribute values was defined in [7]. Based on this case, Chen et al. proposed an incremental algorithm for updating the approximations of RST [7]. Then, they also proposed an incremental approach for update approximations of DRSA in incomplete information systems [8]. Furthermore, they presented matrix-based incremental algorithms for updating decision rules in case of attribute values' coarsening and refining [6].

In fact, the attribute values often evolve over time which may lead some or all of obtained knowledge to failure. The study of the variation of attribute values is paid close attention in many fields. For example, trading portfolios at financial institutions are typically driven by a large number of financial variables and these variables often exhibit by time-varying volatilities [1]. In seismic risk assessment, the results can be updated every time when new information related to the hazard, seismic