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Dissipativity analysis of memristor-based complex-valued neural networks with time-varying delays ☆

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ABSTRACT

In this paper, the problem of dissipativity analysis for memristor-based complex-valued neural networks (MCVNNs) with time-varying delays is investigated extensively. Dissipativity analysis is an important concept in control system theory and its applications. The analysis in the paper employ results from the theory of differential equations with discontinuous right-hand side as introduced by Filippov. By using the framework of Filippov solution, differential inclusion theory, an appropriate Lyapunov–Krasovskii functional and linear matrix inequality (LMI) technique, several new sufficient conditions for global dissipativity, global exponential dissipativity and strictly $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipativity are derived in the form of complex-valued as well as real-valued LMIs. Both real and complex-valued LMIs guarantee feasibility results for addressed MCVNNs. These LMIs can be solved by using standard available numerical packages. Moreover, the global attractive sets which are positive invariant are obtained. Finally, three numerical examples are established to illustrate the effectiveness of the proposed theoretical results.

1. Introduction

In recent years, the qualitative analysis of complex-valued neural networks (CVNNs) have become a greatly focused and emerging area of research due to the fact that most of the successful applications of neural networks involve complex-valued signals. CVNNs have found widespread applications in various fields such as optoelectronics, filtering, imaging, speech synthesis, computer version, remote sensing, quantum devices, spatiotemporal analysis of physiological neural devices and systems, and artificial neural information processing, see [3,17,18,29] and references therein. It is well known that CVNNs are the extension of real-valued neural networks and they consists of complex-valued states, complex-valued connection weights and complex-valued activation functions. Therefore, the process information of CVNNs is in the complex plane. The main focal point of CVNNs is expected at exploring new capabilities and higher performance and it has been used to solve problems which their real-valued counterparts cannot solve. In CVNNs, activation function plays a very important role. The activation function is usually chosen to be smooth and bounded for real-valued neural networks, whereas it is not the case in CVNNs since we have by Liouville's theorem [27] that, every bounded entire function in the complex domain must be a constant. This is not suitable. Therefore, choosing a suitable activation function is the main challenge in complex-valued

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neural networks. In general, CVNNs have dissimilar and much more difficult properties compared with the real-valued neural networks. Thus, the study of stability analysis of CVNNs have received inevitable interest in the network community. Moreover, some of the complex-valued neural network models have been extensively investigated and many interesting results have been obtained in the existing literature see [6,7,13,19,46,48,49] and references therein.

In hardware implementation of neural networks, time delay is an unavoidable factor due to finite switching speed of the amplifiers and communication time. The existence of time delay may affect dynamic behaviors such as oscillation, instability, divergence, chaos or other poor performance of neural networks [8]. Therefore, the study of stability analysis of neural networks with time-delays has received much more attention and increasing interest in the neural network community. However, many interesting results have been obtained in the literature for the stability analysis of delayed neural networks, see [1,9,10,37] and reference therein. In [1], author has obtained sufficient conditions for the existence, uniqueness and global robust stability of the equilibrium point for Hopfield-type delayed neural networks. By constructing a Lyapunov functional and using matrix-norm inequality, some sufficient conditions have been derived for the existence, uniqueness and global robust stability of interval neural networks with time delays in [9]. Moreover, in the existing literature, many authors have analyzed CVNNs with time delays and some new sufficient conditions have been presented, see [6,13,19,48,49] and references therein.

In electronic circuit theory, memristor is one of the two-terminal nonlinear circuit device. It is well known that, resistors, capacitors and inductors are the three basic circuit elements in electronic circuit theory and these three basic circuit elements are defined in terms of the relationship between two of the four fundamental circuit variables namely, current (i), voltage (v), charge (q) and magnetic flux (φ). Among all these, the connection between the circuit variables charge and flux was missing. The missing connection has been formulated theoretically in the year 1971 by Chua [11]. This element was named as memristor and is disguised from other three basic elements. The value of memristor is not unique; it depends on the voltage applied to the corresponding state. The researchers in Hewlett–Packard Laboratories announced that they had discovered a memristor device based on nanotechnology [32,36]. The main feature of memristor is that it carries a memory of its past, that is, when voltage has been turned off in the circuit, the memristor will remember its most recent value until the voltage is turned on. Also memristor is a useful tool for low-power computation, booting free computer and non-volatile memory storage. Moreover, we can use memristor to build a new model of neural networks to emulate the human brain and their potential application is in next generation computer and brain-like neural computer [30,47]. Based on these features, memristor-based neural networks have received more attention and created increasing interest among researchers. Some interesting results on stability, passivity and dissipativity of memristor-based recurrent neural networks have been proposed in [4,5,15,20,38–43,50–52] and references therein.

Dissipative theory for dynamical systems was first initiated in [44], which has been generalized and extensively investigated for nonlinear systems in [21]. Dissipativity theory provides a fundamental framework for the analysis and design of control systems using input–output description based on system energy related considerations. Dissipativity theory is an important idea which has been used in many areas of sciences and control engineering. This provides strong connection between physics, system theory and control engineering. Dissipativity has proven to be essential and very useful tool for control applications like robotics, active vibration damping, electromechanical systems, combustion engines, circuit theory, and for control techniques like adaptive control, nonlinear H^{∞}, and inverse optimal control [23]. The main idea behind this is that many important physical systems have certain input–output properties related to the conservation, dissipation and transport of energy. Therefore, dissipativity analysis of the dynamical systems has become an active area of research in both theoretical and practical point of view. The study of qualitative analysis of dynamical system will be mainly focused on the stability of equilibrium points of the considered system. But, all the trajectories of neural networks will not approach a single equilibrium point and also we cannot say that every neural networks will have an equilibrium point. In some situations they may not have equilibrium points. In those situations, the concept of dissipativity plays a key role, see [22]. In fact, the idea of dissipativity for dynamical systems is a generalization of stability in Lyapunov sense including many dynamical behaviors such as stability, periodicity and chaos. The main advantage of dissipativity theory is to study complex systems. In dissipativity analysis, we need to find global attractive sets, which are positive invariant. Once it has been found, we can calculate a rough equilibrium point, periodic states and chaotic attractors for the considered dynamical system. Thus, dissipativity analysis has attracted much attention from the researchers, scientists and control community in recent years and many excellent results have been proposed regarding the dissipativity of delayed neural networks, stochastic neural networks see [2,12,16,24,25,28,33,34,45] and references therein.

To the best of our knowledge, global dissipativity, global exponential dissipativity and $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipativity of memristor-based complex-valued neural networks with time-varying delays has not been investigated yet in the existing literature. Inspired by the above discussions, in this paper we study the problem of dissipativity analysis of memristor-based complex-valued neural networks with time-varying delays. Some sufficient conditions are derived to guarantee global dissipativity, global exponential dissipativity and $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipativity of memristor-based neural networks in the form of both complex-valued and real-valued LMIs by using an appropriate Lyapunov–Krasovskii functional, linear matrix inequality (LMI) technique, differential inclusions theory and Filippov solution techniques.

This paper is organized as follows. Some of the existing results are discussed related to our problem in Section 2. Some preliminary Definitions and Lemmas are presented in Section 3. The system description of our problem is presented in Section 4. In Section 5, several sufficient conditions for dissipativity of memristor-based complex-valued neural networks