Information Sciences 349–350 (2016) 119–136

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Complex dynamic behavior of a rumor propagation model with spatial-temporal diffusion terms

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ARTICLE INFO

Article history:
Received 18 October 2014
Revised 11 February 2016
Accepted 16 February 2016
Available online 24 February 2016

Keywords:
Delay
Online social networks
Reaction-diffusion equations
Rumor propagation
Stability

ABSTRACT

Rumor propagation as a typical form of social communication in online social networks has had a significant negative impact on a harmonious and stable society. With the rapid development of mobile communication equipments, traditional rumor propagation models, which depend on ordinary differential equations (ODE), may not be suitable for describing rumor propagation in an online social network. In this paper, based on reaction-diffusion equations, we propose a novel epidemic-like model with both discrete and nonlocal delays for investigating the spatial-temporal dynamics of rumor propagation. By analyzing the corresponding characteristic equations of this model, the local stability conditions of a boundary equilibrium point and a positive equilibrium point are established. By applying the linear approximation method of nonlinear systems, sufficient conditions are derived for the existence of Hopf bifurcation at the above two kinds of equilibrium points. Moreover, a sensitivity analysis method based on the density of spreading users is proposed, and then in theoretical and experimental aspect we identify some sensitive parameters in the process of rumor propagation. Finally, numerical simulations are performed to illustrate the theoretical results.

1. Introduction

Online social networks, such as Facebook, Twitter, Digg, the blogosphere and so on, allow hundreds of millions of Internet users worldwide to produce and consume content [1,7,12,13,16,25,38]. Such networks provide access to a vast information space on an unprecedented scale. With the popularization of online social networks, more and more Internet users, for their own purposes which may include diverting attention, manufacturing momentum, causing panic, harming national honor and interests and so on, have begun to spread harmful information through online social networks. For example, in the past decade, rumor propagation not only in small-world networks [4,39,40] but also in scale-free networks [21,22] has attracted much attention from physical and sociological research communities [6,11,17]. Rumor is an important form of social communication, and its propagation plays a significant negative role in a harmonious and stable society. Hayakawa [8] defines rumor as fact-like but unconfirmed information about some event, which can quickly spread through chains of communication on a large scale. Shibutani [27] regards rumor as collective problem-solving, in which people caught in ambiguous situations try to construe a meaningful interpretation by pooling their intellectual resources. Recently, Zhao [33] considers

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http://dx.doi.org/10.1016/j.ins.2016.02.031
0020-0255/

that rumor is a kind of visionary, mostly based not on objectivity but on subjective willingness and fabricated message. Usually, rumor involves influential events such as political and economic issues or public interesting news. Compared with the word-of-mouth way of rumor propagation of past times, rumor propagation in online social networks has become more flexible, faster, and more destructive. Convenient and efficient means to spread rumor may cause panic as well as economic loss resulting from the accompanying unexpected events [28]. These seriously threaten the stability of the society.

To reduce or eliminate the danger of the rumor propagation in online social networks, it is necessary to adequately understand the dynamic characteristics of rumor propagation. Mathematical modeling, an efficient tool, has been used to reveal many phenomena, including the dissemination of gossip, rumors or information. To the best of our knowledge, research engaging rumor propagation models started during the 1960s. In 1964, Daley and Kendall [2,3], divided agents into three classes: *ignorants*, *spreaders*, and *stiflers*, and then they proposed the first classical rumor propagation model, which was called DK model by the later scholars. In fact, the DK model together with its variants, such as the Maki–Thompson (MK) model [20], are conceptually similar to the classical SIR (susceptible-infected-recovered) epidemic models. Given the similarities of rumor propagation and epidemic diffusion, many researchers since then have used the epidemiological model to describe information or rumor propagation on the complex networks [10,18,19,34,35,44–47,49]. Zanette, based on static [40] and dynamic [39] small-world networks, respectively provided simulations of the deterministic MK model. Later, by means of Monte–Carlo simulations and numerical solutions of a set of mean-field equations, Moreno et al. [21,22] considered the stochastic version of the MK model on scale-free networks. Recently, references [10,46] based on utility theory, both proposed a simple model to analyze the interplay between rumor propagation and authorities' actions in emergency situation. Zhao et al. [44,45] provided a detailed description of rumor propagation by establishing a SIR epidemic-like model and a so-called SIHR model. Specifically, they considered the forgetting mechanism in this work. Further, combined with the network structure, Zhao et al. [47] discussed a rumor propagation model using mean-field equations with a variable forgetting rate. Their results showed that the final size of rumor propagation is much larger under a variable forgetting rate compared to that under a constant forgetting rate.

The existing models of rumor propagation in online social networks have concentrated only on the temporal dimension. Recently, a diffusive logistic (DL) model with a spatial-temporal diffusion term was proposed in [31] to study the information propagation process in online social networks. The authors described the spatial distance by using a new concept, friendship hops, and abstractly divided the information diffusion process in online social networks into two separate processes: a growth process and a social process. Wang et al. in [32] based on a partial differential equation (PDE), further proposed a linear diffusive model to understand the information diffusion process over both temporal and spatial dimensions. Combined with the empirical observations in the Digg data set, they proved the performance of the proposed linear diffusive model. Following Wang et al.; Zhu et al. [51] developed a PDE mathematical modeling with consideration of a delayed feedback controller to effectively control the diffusion of harmful information in online social networks. To our knowledge, considering PDE rumor propagation model in online social networks is still at the preliminary stage. Therefore, these spatial-temporal models will provide a new insight to for studying rumor propagation in online social networks.

It is worth noting that most of the above-mentioned works on rumor propagation modeling assume that there is no time delay over rumor propagation. In fact, in emergency situations, a government sometimes cannot promptly take notice of the perniciousness of rumors [51]. Thus, the interplay between rumor diffusion and authorities' actions in an emergency situation is not always synchronized. That is, we should assume that a time delay exists before the authorities' actions on rumor diffusion. Further, as in epidemic models [15,26,52], when studying the process of rumor propagation we should consider that there exists an incubation period before an influenced ignorant user can or will spread rumors. In this paper, our fundamental purpose is to present a novel rumor propagation model with more realistic significance in theory and further analyze the dynamic characteristic of this model in mathematics. Our main contributions and advances are summarized as follows.

(i) Improvements on the previous rumor propagation models. The deterministic epidemic-like rumor propagation models proposed for online social networks in the majority of existing literatures are mainly based on ODE [24,30,35,42,45], which deals only with collective social processes over time without considering space factors for mobile online social networks. Though some researchers recently have begun to study both temporal and spatial patterns of information diffusion in social media by PDE [31,32,51], they may ignore the spatial-temporal delay phenomenon in information transmission. In our work, based on the theory of partial function differential equations (PFDEs), we develop a spatial-temporal epidemic-like rumor propagation model with the discrete and nonlocal delays, which makes up perfectly for the deficiencies of the previous literatures.

(ii) Innovations about parameter identification of a rumor propagation model. As is well known, many factors, such as the spreading rate, the recovery rate, the forgetting rate, the average degree of the network and so on, may affect the evolution of a dynamical process. Thus, distinguishing the sensitivity of these parameters is significant for understanding the mechanism of rumor propagation in online social networks. However, in most of the previous literatures, scholars randomly selected a parameter and then varied the values of this parameter to give a numerical simulation about their proposed models [23,24,44,45]. In this work, by applying sensitivity analysis in mathematics, we study the relationship between the density of spreading users and the parameters in our proposed model. This provides a new to research rumor propagation and makes up for the deficiencies in the selection of parameters for the previous literatures.