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Solution sets of finite fuzzy relation equations with sup–inf composition over bounded Brouwerian lattices ☆

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ABSTRACT

This paper considers the resolution of finite fuzzy relation equations with sup–inf composition over a bounded Brouwerian lattice. The solution sets of finite fuzzy relation equations on a bounded Brouwerian lattice are described in a similar way as those of linear spaces of n-dimensional vectors in linear algebra.

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1. Introduction

The study of fuzzy relation equations is one of the most appealing subjects in fuzzy set theory, both from a mathematical and a systems modelling point of view (see [6]). In 1976, Sanchez introduced the fuzzy relation equations with sup–inf composition (see [21]). Then several authors have further enlarged the theory with many papers (see [9,15] for an extensive bibliography). Among them, Higashi et al. [12] proved that the solution set of a finite fuzzy relation equation over the unit interval [0,1] can be determined by finite number of minimal solutions and the greatest solution, since then, many works about these kinds of equations focus on finding a more simple algorithm to calculate all minimal solutions (see e.g. [14,15,27]). In 2002, Chen et al. [3] proved that the problem of solving finite fuzzy relation equations over the unit interval [0,1] is an NP-hard problem in terms of computational complexity. On a finite fuzzy relation equation with sup–inf composition assigned over a Brouwerian lattice, Zhao [30] determined its entirely solution set, De Baets [6] constructed all minimal solutions and Wang [24,25] showed that every solution has a minimal solution and gave a formula of the number of minimal solutions if its right-hand side has irredundant finite decomposition into join-irreducible elements. There are also other papers which discussed the topic on solving fuzzy relation equations with different composite operators over various lattices (see e.g. [7,11,13,16–20,22,23,26]. In particular, Zhang et al. gave the solution of matrix equations in distributive lattices and studied the problem of solving a finite relation equation with sup-conjunctor composition over a complete lattice in 1991 and 2008, respectively (see [28,29]). Compared with linear algebraic systems, fuzzy relation equations are just such equations which replace the plus-product composition by sup–inf composition and replace the field with a lattice. Therefore, it is a natural idea that whether we can solve them in a similar way as those of linear algebraic systems. In fact, as algebraic structure, a linear (vector) space is a special case of a module over a ring, i.e. a linear space is a unitary module over a field, and a bounded Brouwerian lattice is a commutative semiring (see [1,10]). With these in mind, this paper investigates a fuzzy

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relation equation with sup–inf composition over a bounded Brouwerian lattice $(L, \vee, \wedge, 0, 1)$ and describes its solutions with n-dimensional vectors. Let $\underline{n} = \{1, \ldots, n\}$ be the set of the first n natural numbers, and let $(a_1, a_2, \ldots, a_n)^T$ denote the transpose of $(a_1, a_2, \ldots, a_n)$, i.e. a column vector. Then the fuzzy relation equation is defined as follows

$$A \odot \mathbf{x} = \mathbf{b}, \tag{1}$$

where $\odot$ is the sup–inf composition, $\mathbf{x} = (x_i)^T_{i \in \underline{n}}$ is unknown, $A = (a_{ij})_{m \times n}$ and $\mathbf{b} = (b_i)^T_{i \in \underline{m}}$ are known with $a_{ij}, b_i \in L$, i.e.,

$$(a_{i1} \wedge x_1) \vee \cdots \vee (a_{in} \wedge x_n) = b_i, \quad i \in \underline{m}.$$

Let $\mathcal{X}_1 = \{\mathbf{x} : A \odot \mathbf{x} = \mathbf{b}\}$.

This paper is organized as follows. For the sake of convenience, some notions and previous results are given in Section 2. Sections 3 and 4 are due to describe the solution sets of fuzzy relation equations over the unit interval [0,1] and bounded Brouwerian lattices in a view of semilinear spaces, respectively. Conclusions are given in Section 5.

2. Previous results

In this Section, we give some definitions and preliminary lemmas.

Definition 2.1 (*Zimmermann [32] and Golan[10]*). A semiring $\mathcal{L} = \langle L, +, \cdot, 0, 1 \rangle$ is an algebraic structure, such that

(i) $(L,+,0)$ is a commutative monoid,
(ii) $(L,\cdot,1)$ is a monoid,
(iii) $r \cdot (s + t) = r \cdot s + r \cdot t$ and $(s + t) \cdot r = s \cdot r + t \cdot r$ hold for all $r,s,t \in L$,
(iv) $0 \cdot r = r \cdot 0 = 0$ holds for all $r \in L$,
(v) $0 \neq 1$.

A semiring is *commutative* if $r \cdot r' = r' \cdot r$ for all $r, r' \in L$.

Example 2.1 (*Zhao et al. [31]*). The fuzzy algebra [0,1] under the operations $a + b = \sup\{a,b\}$ and $a \cdot b = \inf\{a,b\}$, the nonnegative real numbers with the usual operations of addition and multiplication, the nonnegative integers under the operations $a + b = \text{g.c.d.}\{a,b\}$ and $a \cdot b = \text{l.c.m.}\{a,b\}$, where $a,b \in L$ (where L is the set of all the nonnegative integers) and g.c.d. (resp. l.c.m.) stands for the greatest (resp. smallest) common divisor (resp. multiple) between a and b, are all commutative semirings with 0,1.

The following definition of a semimodule is taken from Golan [10].

Definition 2.2. Let $\mathcal{L} = \langle L, +, \cdot, 0, 1 \rangle$ be a semiring. A left semimodule is a commutative monoid $\mathcal{A} = \langle A, +_A, 0_A \rangle$ for which an external multiplication $L \times A \to A$, denoted by ra, is defined and which for all $r, r' \in L$ and $a, a' \in A$ satisfies the following equalities:

(i) $(r \cdot r')a = r(r'a)$,
(ii) $r(a +_A a') = ra +_A ra'$,
(iii) $(r + r')a = ra +_A r'a$,
(iv) $1a = a$,
(v) $0a = r0_A = 0_A$.

The definition of a *right semimodule* is analogous, where the external multiplication is defined as a function $A \times L \to A$.

Definition 2.3. Let $\mathcal{L} = \langle L, +, \cdot, 0, 1 \rangle$ be a semiring. Then a semimodule over $\mathcal{L}$ is called a semilinear space.

Note that in Definition 2.3, a semimodule stands for a left $\mathcal{L}$-semimodule or a right $\mathcal{L}$-semimodule as the same as that of [8]. The notion of a semilinear space first appeared in [16] in connection with power algebras over semirings, it has been used later in [8] to explain fuzzy systems and their principles. Elements of a semilinear space will be called *vectors* and elements of a semiring *scalars* (called also coefficients). The former will be denoted by bold letters to distinguish them from scalars.

Without loss of generality, in what follows, we consider left $\mathcal{L}$-semimodules for convenience of notation.

Example 2.2. Let $\mathcal{L} = \langle L, +, \cdot, 0, 1 \rangle$ be a semiring, $V_n(L) = \{(a_1, a_2, \ldots, a_n)^T : a_i \in L, i \in \underline{n}\}$. Define

$$\mathbf{x} + \mathbf{y} = (x_1 + y_1, x_2 + y_2, \ldots, x_n + y_n)^T,$$
$$r\mathbf{x} = (r \cdot x_1, r \cdot x_2, \ldots, r \cdot x_n)^T$$

for $\mathbf{x} = (x_1, x_2, \ldots, x_n)^T$, $\mathbf{y} = (y_1, y_2, \ldots, y_n)^T \in V_n(L)$ and $r \in L$. Then $\mathcal{V}_n = \langle V_n(L), +, 0_{n\times1} \rangle$ is a semilinear space over $\mathcal{L}$ with the zero element $0_{n\times1} = (0, 0, \ldots, 0)^T$. Similarly, we can also define the operations of addition and external multiplication on row

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