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A note on dynamic relational systems

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ABSTRACT

It is known that weak and strong definabilities are defined for multiple-source approximation systems by Khan and Banerjee. This paper presents, in a more general setting a discussion on definabilities of sets in dynamic relational systems. We prove that the inverse-image of a weak or strong definable set with respect to relation preserving function is also weak or strong definable set, respectively. On the way, we show that the inverse-image of a reduct of attribute set is also a reduct under the object function of an information system homomorphism. Further, we give the connections between definable sets and the topology determined by the intersections of the topologies of reflexive relations. A quasi-uniformity is a filter on the cartesian product of a given universe satisfying certain conditions. In fact, every quasi-uniformity is a dynamic relational system where the relations are reflexive. In this respect, we discuss on the connections between approximation systems and (quasi) uniformities.

1. Introduction

Pawlak's rough set theory is a useful extension of set theory for information systems determined by a source [18,19]. Pagliani defined a dynamic relational system which is a pair $(U, \{r_i\}_{i\in I})$, where $\{r_i\}_{i\in I}$ is a family of arbitrary relations on U and I is an arbitrary index set. He studied on pre-topological spaces corresponding dynamic relational systems [16,17]. Recently, Khan and Banerjee discussed on multiple-source approximation systems as a special case of dynamic relational systems where the index set I is countable [11–13]. Further, M-indiscernibility spaces studied by Juan Lu et al. is also a dynamic relational system where the relations are equivalence [14]. In fact, dynamic nature of an information system depends on the attributes related to objects of the universe. An information system can be changed adding or removing some attributes which provide the relations representing the information systems, and this leads to dynamic systems having different approximations. In [26], dynamic relational systems of attributes of multiple different types are considered, and lower and upper approximations are defined using composite relations. In [15], maintaining approximations dynamically are considered in set-valued ordered decision systems under the attribute generalizations. In [2], the change of attribute domain are considered and the alteration of knowledge granulation with respect to the variation of data sets are discussed.

On the other hand, approximation operators and definability are the core concepts of rough set theory [4,5,9,11–13,20,22]. Essentially, weak lower approximation, weak upper approximation, strong lower approximation and strong upper approximation of a set defined in [11–13] can be easily considered for dynamic relational systems. In this work, we show that the inverse-image of a weak or strong definable set with respect to relation preserving function is also a weak or strong definable set, respectively.

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Recall that an information system is a quadruple $S = (U, AT, V, f)$ where U is a set of objects, AT is a set of attributes, $V = \bigcup_{a \in AT} V_a$ is a set of values of attributes and V_a is the domain of a where $f : U \times AT \to V$ is a description function such that $f(x, a) \in V_a$ for every $x \in U$ and $a \in AT$ [6,7,24]. Now let $S = (U, AT, V, f)$ be an information system, $B \subseteq AT$ and $x, y \in U$. The equivalence relation

$$IND(B) = \{(x, y) | \forall a \in B,\ f(x, a) = f(y, a)\}$$

is called B-indiscernibility relation. Set $[x]_B = \{y \in U | (x, y) \in IND(B)\}$. A subset $A \subseteq AT$ is called reduct of AT, if it satisfies the following conditions [20]:

(1) $IND(A) = IND(AT)$,

(2) $\forall B \subset A,\ IND(B) \neq IND(AT)$.

An information system homomorphism [7] of S into S' where $S' = (U', AT', V', f')$ is a triple $h = (h_O, h_A, h_D)$ where h_O is a mapping of U into U', h_D is a mapping of V into V' and h_A is a mapping of AT into AT' if for all $x \in U$ and $a \in AT$,

$$h_D(f(x, a)) = f'(h_O(x), h_A(a)).$$

We prove that if $h = (h_O, h_A, h_D)$ is an information system homomorphism of $S = (U, AT, V, f)$ into $S' = (U', AT', V', f')$ and B' is reduct of $h_A(AT)$ in S', then $h_A^{-1}(B')$ is a reduct of AT in S. Then we give the connections between definable sets and the topology determined by the intersections of the topologies of reflexive relations. A quasi-uniformity is a filter on the cartesian product of a given universe satisfying certain conditions. In fact, every quasi-uniformity is a dynamic relational system where the relations are reflexive. Here, we also discuss on the connections between approximation systems and (quasi) uniformities given in [21].

2. Dynamic relational systems

Definition 1. Let U be a set and $\{r_i\}_{i \in I}$ a family of binary relations on U. Then the pair $(U, \{r_i\}_{i \in I})$ is called a dynamic relational system [17]. If I is a countable index set, then the pair $(U, \{r_i\}_{i \in I})$ is called a multiple-source approximation system [13].

Clearly, every multiple-source approximation system is also a dynamic relational system. Now let (U, r) be an approximation space. Recall that the lower and upper approximation of a subset A of U are defined by

$$\underline{apr}_r A = \{x | \forall y \in U,\ (x, y) \in r \Rightarrow y \in A\}, \quad \text{and} \quad \overline{apr}_r A = \{x | \exists y \in U,\ (x, y) \in r \ and\ y \in A\},$$

respectively [23]. For the sake of shortness, we use the notation $\underline{A}_r$ for $\underline{apr}_r(A)$ and $\overline{A}_r$ for $\overline{apr}_r(A)$.

Let $(U, \{r_i\}_{i \in \mathbb{N}})$ be a multiple-source approximation system (MSAS) and $X \subseteq U$. The strong lower approximation $\underline{X}_s$, weak lower approximation $\underline{X}_w$, strong upper approximation $\overline{X}_s$ and weak upper approximation $\overline{X}_w$ of X, respectively, are defined as follows.

Definition 2 [12].

$$\underline{X}_s = \bigcap_{i \in I} \underline{X}_{r_i}, \quad \underline{X}_w = \bigcup_{i \in I} \underline{X}_{r_i}$$
$$\overline{X}_s = \bigcap_{i \in I} \overline{X}_{r_i}, \quad \overline{X}_w = \bigcup_{i \in I} \overline{X}_{r_i}$$

For the above approximations we have the following inclusions [12]:

$$\underline{X}_s \subseteq \underline{X}_w \subseteq X \subseteq \overline{X}_s \subseteq \overline{X}_w.$$

Definition 3 [12].

(i) X is said to be *lower definable*, if $\underline{X}_s = \underline{X}_w$.
(ii) X is said to be *upper definable*, if $\overline{X}_s = \overline{X}_w$.
(iii) X is said to be *strong definable*, if $\underline{X}_s = \overline{X}_w$.
(iv) X is said to be *weak definable*, if $\overline{X}_s = \underline{X}_w$.

Recall that [1] if (U, r) and (V, h) are any two approximation spaces, then a function $f : (U, r) \to (V, h)$ is called *relation preserving* if

$$\forall u, u' \in U,\ (u, u') \in r \Rightarrow (f(u), f(u')) \in h.$$

This concept can be considered for dynamic relational systems as in [14]: