Information Sciences 293 (2015) 351–369

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

An efficient bi-convex fuzzy variational image segmentation method ☆

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ARTICLE INFO

Article history:
Received 8 December 2013
Received in revised form 25 August 2014
Accepted 12 September 2014
Available online 22 September 2014

Keywords:
Image segmentation
Computational intelligence
Fuzzy logic
Partial differential equation

ABSTRACT

Image segmentation is an important and well-known ill-posed inverse problem in computer vision. It is a process of assigning a label to each pixel in a digital image so that pixels with the same label have similar characteristics. Chan–Vese model which belongs to partial differential equation approaches has been widely used in image segmentation tasks. Chan–Vese model has to optimize a non-convex problem. It usually converges to local minima. Furthermore, the length penalty item which is critical to the final results of Chan–Vese model makes the model be sensitive to parameter settings and costly in computation. In order to overcome these drawbacks, a novel bi-convex fuzzy variational image segmentation method is proposed. It is unique in two aspects: (1) introducing fuzzy logic to construct a bi-convex object function in order to simplify the procedure of finding global optima and (2) efficiently combining the length penalty item and the numerical remedy method to get better results and to bring robustness to parameter settings and greatly reduce computation costs. Experiments on synthetic, natural, medical and radar images have visually or quantitatively validated the superiorities of the proposed method compared with five state-of-the-art algorithms.

1. Introduction

Image segmentation is a classical issue in computer vision. It is a process of assigning a label to each pixel in a digital image so that pixels with the same label have certain similar characteristics. State-of-the-art methods include thresholding [15], clustering [1,12,17,24], graph cuts [31], region split and merge [9], matrix decomposition based methods [18,33,46] and partial differential equation based methods [4,6,7,16,37,38,41,42]. Since image segmentation is an ill-posed inverse problem [10], computational intelligence (CI) techniques including evolutionary computation [21,28], neural networks [8] and fuzzy logic [12,17,23,24] have been successfully adopted to solve it.

Besides CI techniques, partial differential equation (PDE) approaches have been intensively developed in image segmentation since 1990s. One of the most classical models belonging to this category is Chan–Vese (CV) model [7]. However, CV model has three main drawbacks: (1) converging to local optima [3], (2) being sensitive to selection of parameters [7] and (3) computational inefficiency [23].

☆ This work was supported by the National Natural Science Foundation of China (Grant Nos. 61273317, 61422209, 61473215), the National Top Youth Talents Program of China, the Specialized Research Fund for the Doctoral Program of Higher Education (Grant No. 20130203110011) and the Fundamental Research Fund for the Central Universities (Grant Nos. K50510020001 and K5051202053).

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http://dx.doi.org/10.1016/j.ins.2014.09.023

Intuitively, there are two ways to overcome the first drawback by introducing CI techniques. One can use evoluationary computation to find approximate global optima or introduce fuzzy logic to construct a bi-convex objective function by introducing fuzzy logic. As computation cost may be dramatically increased by adopting evolutionary computation, in this paper, we follow the latter thought so that gradient descent method, a fast method converging to local optima, can be used.

The most appealing feature of CV model is its robustness to noises because it contains a length penalty item. However, this item also brings potential danger. That is, if the weight of this item is too large, the algorithm will fail to converge. Unfortunately, after a deep investigation, we found it is difficult to determine manually or automatically whether the weight is too large or not. What's more, the computation of this item is too costly.

Our proposed method to solve the last two drawbacks is inspired by re-initialization [29] and its efficient alternatives (regularizations) [25,26,39,44]. They are called numerical remedy method. These methods are designed to keep the stability during the iterative solution of PDEs. Finding that numerical remedy methods and the length penalty item have similar characteristics, we propose a new way to efficiently combine them, which gives better results, reduces the computation costs and brings robustness to parameter settings.

Our main contributions are (1) introducing fuzzy logic to CV model to form a bi-convex object function and (2) efficiently combining numerical remedy methods and the length penalty item to give better results, reduce the computation costs and bring robustness to parameter settings. The proposed method is tested on different kinds of images to validate its effectiveness by comparing with five state-of-the-art PDE models [23,25,26,39,44]. Results show that our model outperforms the compared models in both segmentation results and computation costs.

The rest of this paper is organized as following. Section 2 gives the motivation of our work. Section 3 describes the proposed method in detail. In Section 4, experimental results are given to validate the effectiveness of the proposed model. The last section presents our concluding remarks.

2. Motivations

Let Ω be a bounded closed subset of $\mathbf{R}^2$, which is the image domain. Let I: $\Omega \to \mathbf{R}$ be a given image. Let c_1 and c_2 be two centers. Our goal is assigning pixels of I into two segments belonging to c_1 and c_2 respectively. As mentioned in Section 1, the major work of this study lies in the following two aspects: (1) a novel fuzzy energy functional and (2) elegant combination of numerical remedy methods and the length penalty item. Our detailed motivations are as follows.

2.1. Motivation of defining an effective convex or bi-convex energy functional

The CV model [7] gives that

$$E_{CV} = \mu|\Gamma| + \lambda_1 \int_{\text{inside}(\Gamma)} (I - c_1)^2 dx + \lambda_2 \int_{\text{outside}(\Gamma)} (I - c_2)^2 dx, \tag{1}$$

where c_1 and c_2 are the average values of a given image I inside and outside the curve Γ, respectively, and $\mu > 0$, λ_1, $\lambda_2 > 0$ are weights for the length penalty item and the fitting item, respectively. Or equivalently, Eq. (1) can be rewritten as:

$$E_{CV} = \mu|\Gamma| + \lambda_1 \int_{\Omega} H(\phi)(I - c_1)^2 dx + \lambda_2 \int_{\Omega} (1 - H(\phi))(I - c_2)^2 dx, \tag{2}$$

where $H(\phi)$ is the Heaviside function that equals 1 when $\phi \geqslant 0$ and equals 0 when $\phi < 0$ and where ϕ is called level set function which acts as an indicator of foreground ($\phi > 0$), boundary ($\phi = 0$) and background ($\phi < 0$).

2.1.1. Drawbacks of Chan–Vese model

Although the existence of global minimum of Eq. (1) or Eq. (2) has been proved, the convergence is still an open problem [2]. Fig. 1 and Table 1 give an example to illustrate the convergence to local optima.

We use the Jaccard similarity (JS) [11] value as the quantitative measure. The JS value is defined as $\text{JS}(S_1, S_2) = |S_1 \cap S_2|/|S_1 \cup S_2|$, where S_2 is the groundtruth data and S_1 is the segmentation results. From Fig. 1 and Table 1, it can be seen that as the weight μ increases, initialization with type I that covers balanced area of foreground and background, it tends to achieve a better result. However, for the initialization with type II that only covers the background, the opposite is right.

Another disadvantage of CV model is that the length penalty item limits the option of initial value of level set function. The option of initial level set function also depends on μ (see Fig. 2 for an example and analysis in Appendix A). Without losing generality, assume ϕ is initialized as

$$\phi = \begin{cases} \phi_0, & \textit{inside curve}, \\ -\phi_0, & \textit{outside curve}. \end{cases} \tag{3}$$

It is known that large ϕ_0 will cause the algorithm take long time to converge. On the other hand, the relaxed lower bound of ϕ_0 is given by

$$\phi_0 \geqslant \frac{\mu}{2\sqrt{2}|(-(I - c_1)^2 + (I - c_2)^2)|}. \tag{4}$$