



Incomplete interval valued fuzzy preference relations



Asma Khalid*, Ismat Beg

Center for Mathematics and Statistical Sciences, Lahore School of Economics, Lahore, Pakistan

ARTICLE INFO

Article history:

Received 7 May 2015

Revised 4 February 2016

Accepted 8 February 2016

Available online 12 February 2016

Keywords:

Interval valued fuzzy preference relation

Incomplete relation

Additive consistency for interval valued relation

Interval valued multiplicative preference

ABSTRACT

An interval valued preference relation is a preference structure that is used to describe uncertainty in complex decision making problems. Retrieving complete information from experts is improbable in real life scenarios. Discarding incomplete information leads to loss of important data. In this paper, we introduce an upper bound condition to deal with incomplete interval valued fuzzy preference relations. With the help of this condition, missing preferences are estimated such that they are expressible. Moreover, the resultant complete relation is consistent. In case if an expert is unable to abide by the proposed upper bound condition, an algorithm is formulated to assist the expert in complying to the upper bound condition.

© 2016 Elsevier Inc. All rights reserved.

1. Introduction

Fuzzy preference relations have successfully modeled decision making problems. To combat vagueness, interval valued preference relations are introduced by Bilgic [7] and Xu [30,33]. Interval valued relations add flexibility to the uncertainty representation problem [3]. Construction of interval valued fuzzy preference relations using ignorance functions and their applications is introduced by Barrenechea et al. [4]. Interval weights were derived by Wang and Kevin [26] using goal programming approach on interval fuzzy preference relations. Application of decision making in finance can be studied in [9].

Literature proposes several methods to incorporate for incompleteness in preference relations [8,10,13]. Some methods discard decision makers providing incomplete information. Others estimate missing values using preferences of other experts. However, methods that consider expert's own preferences to estimate the missing information are more appropriate [5,25].

Herrera-Viedma et al. [14] proposed methods to complete preference relation of an expert providing $(n - 1)$ preference values of the form $\{p_{12}, p_{23}, \dots, p_{(n-1),n}\}$. Furthermore, [13] included the case where a complete row or column of preference intensities is given by the expert. Khalid and Awais [16] stressed that in earlier methods, estimated missing preferences surpassed the domain. Transformation functions were introduced to bring such preferences back to the unit interval but at the cost of voiding the originality of the preference values provided by experts. It was further discussed in [16,17] that the existing methods to complete an incomplete preference relation did not focus on consistency of the resultant relation. Therefore, to estimate missing values that do not surpass the unit interval, an upper bound condition was presented in [16]. The completed preference relation with this condition satisfied additive transitivity and Saaty's consistency in case of incomplete multiplicative fuzzy preference relations.

The focus of this paper is on interval valued fuzzy preference and multiplicative fuzzy preference relations. Given a large set of alternatives, it is reasonable to expect incomplete interval valued preferences intensities from decision makers. The

* Corresponding author.

E-mail address: asmakhalid4444@gmail.com (A. Khalid).

reason for this incompleteness could be lack of information, uncertainty, ambiguity or inability to make a choice. Decision making processes should be modeled in a way such that they incorporate for incompleteness, estimates missing preferences and also promise consistency of the resultant completed relation. An inconsistent preference relation of any nature is less useful as compared to preference expressions that are consistent. Consistency in preference and multiplicative preference relations is addressed in [19–21,23,27]. In interval valued fuzzy preference relations, consistency was introduced by Bilgic [7].

Zeshui [33] introduced incomplete interval fuzzy preference relation and using continuous interval argument ordered weighted average, transformed them into incomplete fuzzy preference relations. Alonso et al. [2] presented a method to estimate missing preference values in fuzzy, multiplicative, interval valued and linguistic fuzzy preference relations. Jiang [15] proposed a similarity index for interval fuzzy preference relations. He checked consistency degree of the group undergoing decision making process by using this similarity index. Genc et al. studied [12,31] and introduced test for consistency of interval valued multiplicative preference relations. They proposed that instead of deriving priority weights from linear programming models to check for consistency, simple formulas can be derived from the concept of interval multiplicative transitivity of an interval fuzzy preference relation. Furthermore, two approaches to estimate missing values in interval valued multiplicative relations were proposed.

This paper aims to incorporate incomplete interval valued fuzzy preference and multiplicative fuzzy preference relations in a decision making process. A complete preference relation consists of $n(n-1)/2$ preference intensities in the upper diagonal of the preference relation. If a decision maker is unable to compare two given alternatives then the situation cannot be modeled as that of indifference.

In this paper we emphasize that instead of deriving weights by linear programming model [31] or calculating the priority vectors to estimate missing values [12], the missing values can be estimated by implying an upper bound condition on the decision makers. Alonso et al. [1] proposed a general method to estimate missing values in case of interval valued fuzzy preference relations. The drawback of this method is that estimated values may come out to be supersets of the unit interval. Such an estimation will not imply anything about the preference of the decision maker. This work is motivated to estimate missing preferences while ensuring their expressibility.

If estimated preferences void the giving domain then the resultant relation will not qualify as an interval valued preference relation. Therefore, such estimated preferences will have no interpretation. Moreover, this paper stresses on the fact that if the surpassed estimated preferences are translated to the appropriate domain using transformation functions, then such a decision will cost originality of the preference intensities provided by the decision makers. Therefore, this paper proposes a method which estimates missing preferences while abiding by the specified domain and does not alter the provided preferences of the decision makers. Moreover, this paper extends the work to cater for situations when decision makers are unable to abide by the upper bound condition. In this situation, an algorithm is designed to revise minimum possible opinions such that the upper bound condition is met and consequently, the estimated preferences are expressible.

The paper is organized as follows. Section 1 provides literature review. Section 2 discusses the basic definitions that are used in the sequel. Section 3 introduces the concept of expressible and non-expressible interval valued preferences. This section further defines an upper bound condition, property (ubc) that is imposed on experts if they are to propose incomplete interval valued preference relations. This section proves that if decision makers abide by property (ubc) then the resultant completed relation is expressible and consistent. Section 4 further focuses on the rare case when a decision maker, despite the instructions, is unable to conform to property (ubc). In this section, a rule is defined to carry out minimal possible revisions in the provided preferences so that the incomplete relation satisfies the condition. This rule is defined so that such an interval valued relation is not discarded and so that most of the appropriate information given by the decision maker is used. This section proposes flow chart, Rule 1, to deal with both the situations where experts respect property (ubc) and the case where they fail to do so. Section 5 deals with incompleteness in interval valued multiplicative fuzzy preference relations and state a corresponding upper bound condition (mubc) for such relations. This section also briefly discusses the case when a decision maker fails to satisfy the proposed condition. Section 6 concerns future work and draws conclusion to this work.

2. Preliminaries

Consider $X = \{x_1, x_2, \dots, x_n\}$, ($n \geq 2$) to be a non-empty set of alternatives.

Definition 1. [32] A fuzzy set A on X is characterized by a membership function $\mu_A: X \rightarrow [0, 1]$ where $\mu_A(x)$ is defined as the degree of membership of element x in fuzzy set A for each $x \in X$.

Definition 2. [6,11,22,23] A fuzzy preference relation P on X is characterized by a function $\mu_P: X \times X \rightarrow [0, 1]$ where $\mu_P(x_i, x_j) = p_{ij}$ indicates the preference intensity with which alternative x_i is preferred over x_j .

- i. According to Orlovsky [18], P is additive reciprocal if for all i, j it satisfies $p_{ij} + p_{ji} = 1$.
- ii. P is additive transitive if $p_{ij} = p_{ik} + p_{kj} - 0.5$ for all i, j, k .

Definition 3. [20–22] Let $A \subset X \times X$ denote a multiplicative fuzzy preference relation, the intensity of preference a_{ij} is measured using a ratio scale, particularly, a 1 – 9 scale. Here, $a_{ij} = 1$ indicates indifference between x_i and x_j and $a_{ij} = 9$ indicates that x_i is absolutely preferred to x_j .

Download English Version:

<https://daneshyari.com/en/article/392339>

Download Persian Version:

<https://daneshyari.com/article/392339>

[Daneshyari.com](https://daneshyari.com)