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A systematic approach for embedding of Hamiltonian cycles through a prescribed edge in locally twisted cubes

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ABSTRACT

The locally twisted cube interconnection network has been recognized as an attractive alternative to the hypercube network. Previously, the locally twisted cube has been shown to contain a Hamiltonian cycle. The main contribution of this paper is to provide the necessary and sufficient conditions for determining a characterization of permutations of link dimensions constructing Hamiltonian cycles in a locally twisted cube. For those permutations, we propose a linear algorithm for finding a Hamiltonian cycle through a given edge. As a result, we obtain a lower bound for the number of Hamiltonian cycles through a given edge in an n-dimensional locally twisted cube.

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1. Introduction

The performance of a multiprocessor computer is significantly determined by its interconnection network. Hypercube network is widely used in interconnection network topology due to its many attractive properties such as regularity, recursive structure, node and edge symmetry, maximum connectivity, and effective routing and broadcasting algorithms [8]. The locally twisted cube is a well-known variant of the classical hypercube proposed by Yang et al. [23] and has been attracting much research interest in literatures since its proposal [10,11,22,19,23].

There are many parallel and distributed algorithms developed using such regular data structure as linear arrays, rings, trees, and meshes. Their implementation on a hypercube-type interconnection network very often requires that a specific topology be mapped into the network. Of those commonly used topologies, many efficient algorithms designed on cycles for solving various algebraic problems and graph problems can be found in [8]. To implement a cycle-structure algorithm on a multiprocessor computer, the processes of the parallel algorithm must be mapped to the nodes of the system such that two adjacent processes in the cycle are mapped to two adjacent nodes of the network. Due to efficiently executing a parallel program, the targeted interconnection network possesses a Hamiltonian cycle, that is, a cycle passes every node of the network exactly once if the number of processes in the cycle-structure parallel algorithm equals the number of nodes of the interconnection network.

The Hamiltonian cycle problem has attracted the interest of many researchers and many interesting results have been proposed in last two decades [1–7,9,11–18,20–22,24]. In this paper, we study the problem of embedding a family of *regularly structured* Hamiltonian cycles passing through a given edge in a locally twisted cube. We only consider a family of Hamiltonian cycles being systematically constructed, characterized by the permutation of link dimensions. We show that not

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every permutation can generate a Hamiltonian cycle as it would in the hypercube. Furthermore, the necessary and sufficient conditions are given to determine a characterization of permutations constructing Hamiltonian cycles in a locally twisted cube. For those permutations, we propose a linear algorithm for finding a Hamiltonian cycle through a given edge; besides, we obtain a lower bound for the number of Hamiltonian cycles passing through a given edge in an n-dimensional locally twisted cube.

The rest of this paper is organized as follows: In Section 2, we are given a formal description of the locally twisted cube and define notations used in this paper, including notation for the permutation of link dimension and the reflected link label sequence. Section 3 presents main results of this paper and proposes a linear desired algorithm. Section 4 gives some concluding remarks.

2. Preliminaries

A topology of an interconnected network is conveniently represented by an undirected simple graph $G = (V, E)$, where $V(G)$ and $E(G)$ are the vertex set and the edge set of G respectively. For graph terminology and notation not defined here we refer the reader to [8]. A walk in a graph is a finite sequence $\omega : \lambda_0, e_1, \lambda_1, e_2, \lambda_2, \ldots, \lambda_{k-1}, e_k, \lambda_k$ whose terms are alternately vertices and edges so, for $1 \leqslant i \leqslant k$, the edge e_i has ends λ_{i-1} and λ_i, thus each edge e_i is immediately preceded and succeeded by the two vertices with which it is incident. In particular, a walk ω is called a path if all internal vertices, λ_i for $1 \leqslant i \leqslant k-1$, of the walk ω are distinct. Both vertices λ_0 and λ_k are called end-vertices of the path ω. For simplicity, the path ω is also denoted by $\lambda_0, \lambda_1, \ldots, \lambda_k$. If $\lambda_0 = \lambda_k$, then ω is called a cycle. A cycle of length l is called a l-cycle. A path (respectively, cycle) traversing each vertex of G exactly once is the *Hamiltonian path* (respectively, *Hamiltonian cycle*).

Let $\{0,1\}^n$ denote the set of all binary strings of length n. For two binary strings x and $y \in \{0,1\}^n$, let $x + y$ denote the (bitwise modulo 2) sum of x and y. For every integer $0 \leqslant i \leqslant n-1$, let b_i denote the binary string $x_{n-1}x_{n-2}\ldots x_0$ with $x_i = 1$ and $x_j = 0$ for all $j \neq i$. For every integer $2 \leqslant i \leqslant n-1$, let B_i denote the binary string $x_{n-1}x_{n-2}\ldots x_0$ with $x_i x_{i-1} = 11$ and $x_j = 0$ for all $j \neq i, i-1$. In addition, let $B_1 = b_1$ and $B_0 = b_0$. As a result, $B_i = b_i$ for $i \leqslant 1$ and $B_i = b_i + b_{i-1}$ for $i \geqslant 2$, moreover, $b_i + b_i = B_i + B_i = 0^n$ where 0^n denote a string consisting of n 0 s.

Definition 1. [23] For $n \geqslant 2$, an n-dimensional locally twisted cube, denoted by LTQ_n, is defined recursively as follow:

(1) LTQ_2 is a graph consisting of four nodes labeled with 00, 01, 10, and 11 respectively, connected by four edges $(00, 01)$, $(00, 10)$, $(01, 11)$ and $(10, 11)$.
(2) For $n \geqslant 3$, LTQ_n is built from two disjoint copies of LTQ_{n-1} according to the following steps. Let $0LTQ_{n-1}$ (respectively, $1LTQ_{n-1}$) denote the graph obtained by prefixing the label of each node in one copy of LTQ_{n-1} with 0 (respectively, 1). Each node $0x_{n-2}x_{n-3}\ldots x_0$ in $0LTQ_{n-1}$ is connected to the node $1(x_{n-2}+x_0)x_{n-3}\ldots x_0$ in $1LTQ_{n-1}$ by an edge.

Fig. 1 shows examples of locally twisted cubes, LTQ_3 and LTQ_4. Either $x = y + b_k$ or $x = y + B_k$ for some $0 \leqslant k \leqslant n-1$ if vertices x and y of LTQ_n are adjacent. Therefore, we call y as the k-neighbor of x and (x, y) is labeled by k; besides, (x, y) is called to be type b if $x = y + b_k$ and type B if $x = y + B_k$.

A path in LTQ_n might be specified by the source vertex and a sequence of labels detailing the edges to be traversed, for example, the path in LTQ_3 detailed as having the source vertex 000 and then following the edges labeled 0-2-1 (also denoted as [0-2-1]) is actually the path 000, 001, 111, 101, also denoted as 000[0—2—1]101, where $001 = 000 + b_0$, $111 = 001 + B_2$, and $101 = 111 + B_1$. Therefore, the sequence $L = [d_0 - d_1 - \cdots - d_{m-1}]$ is called an *Link Label Sequence* in LTQ_n if two adjacent labels are not identical where $d_i \in Z_n$, $Z_n = \{0, 1, \ldots, n-1\}$, for $0 \leqslant i \leqslant m-1$. A walk, $\omega(L, u) = \lambda_0, \lambda_1, \lambda_2, \ldots, \lambda_m$, in LTQ_n can be generated with respect to a given link label sequence L and a given vertex u as follows: $\lambda_0 = u$, and λ_j is the d_{j-1}-neighbor of λ_{j-1} in LTQ_n where $0 \leqslant j \leqslant m$. Thus, this walk $\omega(L, u)$ is also represented as $\lambda_0[L]\lambda_m$.

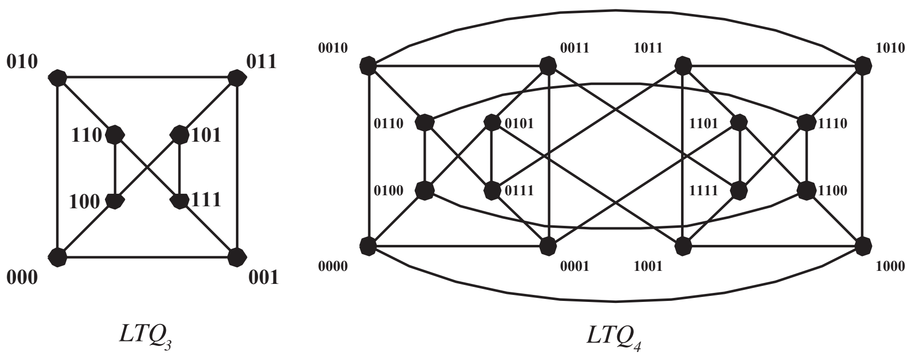

Fig. 1. LTQ_3 and LTQ_4.

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