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Logarithmic least squares prioritization and completion methods for interval fuzzy preference relations based on geometric transitivity

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ABSTRACT

This article introduces the notion of geometric transitivity, which may be used to define consistent interval fuzzy preference relations. Some useful properties are presented for consistent interval fuzzy preference relations. A close investigation reveals that existing methods for determining the analogous property of consistency are not robust to permutations of the pairwise judgments and not able to reflect the hesitancy in the decision-maker's preference. A parametric transformation formula is put forward to convert a normalized interval weight vector into a consistent interval fuzzy preference relation. By minimizing the squared difference between the logarithm of the ratio of the original judgment and the logarithm of the ratio of the converted consistent one, a logarithmic least squares model is developed to derive interval weights from any interval fuzzy preference relation. Based on geometric transitivity, a logarithmic least squares model is established to rectify inconsistency for complete and inconsistent interval fuzzy preference relations, and a logarithmic least squares completion approach is further developed to estimate missing values for incomplete interval fuzzy preference relations. Numerical examples are furnished to show the validity of the proposed models and compare with other existing methods.

1. Introduction

When facing difficulty in directly assigning weights to criteria or sores to alternatives, a decision maker (DM) often uses the pairwise comparison method to elicit his/her preferences in multi-criteria decision making (MCDM). Based on pairwise comparisons, the DM furnishes a multiplicative reciprocal preference relation in the analytic hierarchy process (AHP) [21]. Since preference values in multiplicative reciprocal preference relations are all exact values, the original AHP is unable to deal with vague and uncertain judgments [8–10]. As such, the concept of fuzzy preference relations [19] (also called reciprocal preference relations [6,7,30]) is introduced to characterize DM's judgments with vagueness, and an increasing research interest has been focusing on fuzzy AHP methods [2–9,12,14,15,17–20,23–25,27–36].

Due to complexity and uncertainty of real-world decision problems, it is often a challenge to assign crisp values for DM's pairwise comparison results. It is easier or more natural to assign fuzzy interval numbers for DM's subjective judgments by means of interval fuzzy preference relations [33]. Several interval prioritization methods have been devised for obtaining interval weights from interval fuzzy preference relations. Xu and Chen [34] employ the concept of feasible regions to define

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additive and multiplicative consistency of interval fuzzy preference relations, and develop two linear programs for deriving interval weights from the viewpoints of additive and multiplicative transitivity. Based on Xu and Chen's additive and multiplicative consistency properties, different programming models [24,29] have been established to generate interval priority weights of interval fuzzy preference relations. Xu [32] further puts forward two programming models to rectify an inconsistent interval fuzzy preference relation into the one with additive or multiplicative consistency. Lan et al. [15] present an exchange relation between an additively consistent interval fuzzy preference relation and a multiplicatively consistent interval fuzzy preference relation, and propose a method to obtain interval weights from the viewpoint of multiplicative consistency. Genç et al. [12] put forward a multiplicative transitivity expression to judge whether an interval fuzzy preference relation is multiplicatively consistent or not, and develop a formula to derive interval weights from a consistent preference relation. They also propose two approaches to estimate missing values for incomplete interval fuzzy preference relations. Liu et al. [17] introduce two converted fuzzy preference relations to define a consistent interval fuzzy preference relation, and develop a prioritization method to derive interval multiplicative weights from an interval fuzzy preference relation. Wang and Li [27] adopt interval arithmetic to define additive consistency, multiplicative consistency and weak transitivity of interval fuzzy preference relations, and develop a goal-programming-based framework for deriving interval weights from interval fuzzy preference relations.

The aforesaid research indicates that prioritization and completion methods are generally based on the consistency property of preference relations. A rational consistency property would be conducive to deriving a reasonable and dependable decision result. Aguaron et al. [1] develop a geometric consistency index (GCI) to measure the inconsistency level of a multiplicative reciprocal preference relation, and provide the corresponding thresholds that are analogous to the consistency ratios proposed by Saaty [21]. Recently, Chiclana et al. [4] introduce a functional equation to define cardinal consistency of reciprocal preference relations, and show that multiplicative transitivity proposed by Tanino [23] is an appropriate property to model consistency of fuzzy preference relations. By following this guideline, Xia and Xu [28] adopt two equations to define a perfect multiplicatively consistent interval fuzzy preference relation. However, a close examination reveals that such consistency extension is flawed by not being robust to permutations of the DM's judgments (See a further analysis in Section 3). A similar situation also appears in the consistency definition given by Liu et al. [17], in which two converted fuzzy preference relations with additive consistency are used to define a consistent interval fuzzy preference relation (See a further analysis in Example 3). On the other hand, the multiplicative consistency proposed by Xu and Chen [34] is defined from the viewpoint of feasible regions without directly modeling transitivity among three or more original judgments in a preference relation, implying that such consistency constraint is excessively loose, i.e., too uncertain preferences are usually consistent with others.

This paper focuses on the consistency property and priority method of interval fuzzy preference relations as well as how to estimate missing values for incomplete interval fuzzy preference relations. More specifically, a geometric transitivity definition is first put forward for interval fuzzy preference relations. Some properties are then presented for consistent interval fuzzy preference relations. A geometric mean based uncertainty ratio is introduced to measure the uncertainty level of an interval fuzzy preference relation. Subsequently, a parametric transformation formula is proposed to convert a normalized interval weight vector into a consistent interval fuzzy preference relation, and a logarithmic least squares approach is developed to derive interval weights from an interval fuzzy preference relation. Finally, logarithmic least squares models are put forward to rectify inconsistency and estimate missing values for incomplete interval fuzzy preference relations.

The remainder of the paper is organized as follows. In Section 2, we review consistent fuzzy preference relations and some basic concepts related to interval fuzzy preference relations. Section 3 introduces the geometric consistency definition of interval fuzzy preference relations and provides some useful properties. Section 4 develops a logarithmic least squares model to derive interval weights from interval fuzzy preference relations. Section 5 proposes logarithmic least squares completion and inconsistency rectification methods for interval fuzzy preference relations. Concluding remarks are given in Section 6.

2. Preliminaries

Consider an MCDM problem with a set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$. A fuzzy preference relation [19] (also called reciprocal preference relation [6,7,30]) R on X is denoted by a judgment matrix $R = (r_{ij})_{n\times n}$, where r_{ij} indicates the preference degree of the alternative x_i over x_j such that

$$r_{ij} \in [0,1], r_{ij} + r_{ji} = 1, r_{ii} = 0.5 \text{ for all } i,j = 1,2,\ldots,n \tag{2.1}$$

The greater the value of r_{ij}, the stronger the preference degree of x_i over x_j. If $0.5 < r_{ij} < 1$, then $\frac{r_{ij}}{r_{ji}} = \frac{r_{ij}}{1-r_{ij}} > 1$, indicating that x_i is preferred to x_j with the multiplicative intensity $\frac{r_{ij}}{r_{ji}}$. If $0 < r_{ij} < 0.5$, then $0 < \frac{r_{ij}}{r_{ji}} = \frac{r_{ij}}{1-r_{ij}} < 1$, meaning that x_j is preferred to x_i with the multiplicative intensity $\frac{r_{ji}}{r_{ij}}$. In particular, if $r_{ij} = 0.5$, then $\frac{r_{ij}}{r_{ji}} = \frac{r_{ij}}{1-r_{ij}} = 1$, showing that x_i and x_j are equally preferred.

Definition 2.1 [23]. Let $R = (r_{ij})_{n\times n}$ be a fuzzy preference relation with $0 < r_{ij} < 1$ for all $i, j = 1, 2, \ldots, n$. R is called multiplicatively consistent, if it satisfies