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# A two-stage stochastic mixed-integer program modelling and hybrid solution approach to portfolio selection problems

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ABSTRACT

In this paper, we investigate a multi-period portfolio selection problem with a comprehensive set of real-world trading constraints as well as market random uncertainty in terms of asset prices. We formulate the problem into a two-stage stochastic mixed-integer program (SMIP) with recourse. The set of constraints is modelled as mixed-integer program, while a set of decision variables to rebalance the portfolio in multiple periods is explicitly introduced as the recourse variables in the second stage of stochastic program. Although the combination of stochastic program and mixed-integer program leads to computational challenges in finding solutions to the problem, the proposed SMIP model provides an insightful and flexible description of the problem. The model also enables the investors to make decisions subject to real-world trading constraints and market uncertainty.

To deal with the computational difficulty of the proposed model, a simplification and hybrid solution method is applied in the paper. The simplification method aims to eliminate the difficult constraints in the model, resulting into easier sub-problems compared to the original one. The hybrid method is developed to integrate local search with Branch-and-Bound (B&B) to solve the problem heuristically. We present computational results of the hybrid approach to analyse the performance of the proposed method. The results illustrate that the hybrid method can generate good solutions in a reasonable amount of computational time. We also compare the obtained portfolio values against an index value to illustrate the performance and strengths of the proposed SMIP model. Implications of the model and future work are also discussed.



## 1. Introduction

The essence of portfolio selection problem (PSP) can be described as finding a combination of assets that best satisfies an investor's needs. The theory of PSP was developed by Harry Markowitz firstly in the 1950s. In his work, the PSP was formulated as the mean–variance (MV) model [34], a quadratic optimization model. The basic MV model selects the composition of assets which either achieves a predetermined level of expected return while minimizing the risk, or achieves the maximum expected return within a predefined level of risk.

From a practical point of view, however, the MV model is often considered to be too basic, as it ignores many constraints faced by real-world investors. Presences of real-world trading constraints dramatically increase the complexity of PSP. These constraints include the cardinality constraint (a limit on the total number of assets held in a portfolio), the minimum and/or maximum position size constraint (bounds on the amount of each asset), the minimum trade size constraint (bounds on the

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amount of transaction occurred on each asset) and transaction costs, etc. When such constraints are considered into the basic MV model, the problem is recognized to be NP-complete for the feasibility problem and NP-hard for the optimization problem [5,32].

Along with the trading constraints, another important factor faced by the investors to make a proper investment decision is the market uncertainty. However, in the classic MV model and other models of PSP [9,24,10,28,33], the expected return and covariance between assets are usually based directly on historical data. These models do not account for the uncertainties of the market.

Financial markets are unpredictable and decision should be made with the consideration of market conditions [19,3]. Moreover, investors apply flexible portfolio management strategies by rebalancing portfolios periodically in response to new uncertain market conditions (i.e. changing perceptions regarding the random asset price). Random uncertainties of market, i.e., in terms of asset prices and currency exchange rates, etc. are main factors of the market. Several non-probabilistic uncertainty factors such as vagueness and ambiguity are investigated mainly by fuzzy techniques [15,14,30,46]. In this paper we focus on random uncertainty of the market, i.e., in term of asset prices.

Stochastic programming becomes an increasingly popular technique to model decision making under random uncertainty. It is able to model uncertainties in a flexible way and impose real-world constraints relatively easily [25]. It uses information in the future to make current decisions. Higle and Wallace [18] investigate the uncertainty factors in general stochastic problems. Generally speaking, taking into account of uncertainty helps to improve decision making.

Stochastic programming has been applied to describe a variety of portfolio optimization problems. Models for assetliability and risk management have been proposed in [36,42,41]. New approaches based on stochastic programming for portfolio management have also been proposed [36,12,18,4,13,19,11,42,3,40,41].

In portfolio optimization problem, Gaivoronski et al. [13] investigate the issue of optimal portfolio rebalancing and try to determine whether to rebalance a given portfolio based on the transaction costs and new information of market. Fleten et al. [12] demonstrate how to evaluate stochastic programming models by comparing two different approaches to asset liability management. The first uses multistage stochastic programming, while the other is a static approach. They show that the stochastic programming method dominates the static method to the asset liability management problem due to its ability that adapts the market information.

To solve the stochastic model of PSP, in the literature a wide range of decomposition techniques have been developed [6,35,4,11,40,41]. Birge [6] proposed the nested variant of Benders decomposition. Mulvey and Ruszczyński proposed the augmented Lagrangian decomposition [35]. Stoyan and Kwon [41] proposed a novel decomposition method based on the particular structure of the problem concerned. It decomposes the problem geographically into security and bond sub-problems, which are then further broken into smaller sub-problems.

In this paper, we model the multi-period PSP with a comprehensive set of real-world trading constraints including the cardinality constraint, the minimum position size, the minimum trade size constraint, and transaction costs, etc. In order to model these constraints, we apply a mixed-integer optimization approach. In addition, to account for the uncertainty in the market in term of asset prices to enhance the decision making of investors, we apply stochastic programming to capture future information in the market, i.e. asset prices, represented by a set of random variables. A set of decision variables to rebalance the portfolio is explicitly introduced as the recourse variables in the second stage of stochastic program. These recourse variables are used to amend the first stage decision variables based on the realization of the random variables of asset prices. Thus, the problem is formulated into a two-stage stochastic mixed-integer program.

We set the model in the framework of stochastic programming and formulate the problem in a two-stage setting. This is for simplicity but not primarily. What is more, we see this as an appropriate framework that fulfils the purpose of our investment. The purpose of the model is making current decision under a set of real-world trading constraints while taking into account of future asset price changes. The integral and stochastic nature of the problem makes the two-stage model complex enough. We do not intend to provide tactical rebalance strategy for each future trading but make current decision more flexible with recourse for the future.

We investigate the effect of asset price uncertainty upon the solution to the portfolio optimization problem. Although stochastic programming models have been used for other types of portfolio construction [36,13,19,42,40,41], to our knowledge, this is the first work to develop a multi-period PSP model with a comprehensive set of real-world constraints as well as market uncertainty.

In our previous work [17], initial tests of a hybrid local search method have been conducted on a relatively easier model for deterministic PSPs without uncertainties. In the present work, to address the complex two-stage portfolio selection problem with uncertainty, the model combines integral and stochastic variables. This leads to a more computational challenging SMIP model. Therefore, we adapt a simplification and hybrid method based on our previous method that hybridizes local search with B&B (named LS-B&B) to solve the SMIP *heuristically*. In the LS-B&B, variable fixing together with a local search are applied to generate a sequence of simplified sub-problems. The default B&B search then solves these restricted and simplified sub-problems more easily due to their reduced size comparing to the original one. The idea is to perform computationally inexpensive local search on the surface of certain variables, and then explore the sub-problems by B&B to completion.

In summary, this paper presents two major contributions: (1) a two-stage stochastic mixed-integer program model that formulates a comprehensive set of real-world trading constraints, as well as random uncertainty in the market employing the concepts of scenarios and stages. (2) The application and analysis of an efficient simplification and hybrid solution method to the new two-stage SMIP model, which is proposed and examined for the first time in this work.