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Reduction methods of type-2 uncertain variables and their applications to solid transportation problem

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ABSTRACT

Uncertainty theory is a branch of mathematics for dealing with realistic uncertainties arising out of complexity, changeability, and non-decidability of practical environments. An uncertain variable is defined as a function from the uncertainty space to the set of real numbers and is characterized by an uncertainty distribution. This paper proposes the definition of type-2 uncertain variables within the framework of uncertainty theory through introduction of generalized uncertain measures and focuses on more complex twofold uncertainties. Some uncertainty reduction methods associated with type-2 uncertain variables are also proposed for convenience of applicability, including reduction of optimistic value, pessimistic value and expected value. Moreover, four classes of type-2 uncertain variables are reduced to type-1 uncertain variables with specific uncertainty distributions. Type-2 uncertain optimization methods are applied to solving the fixed charge solid transportation problem with the type-2 uncertain parameters, where the solution methods are also provided for the proposed models. Finally, numerical experiments are implemented to demonstrate application and sensitivity analysis of the proposed approaches.

1. Introduction

Different types of inherent uncertainties often occur in practical decision-making systems, such as randomness, fuzziness, and roughness. To effectively handle these uncertainties in real life situations, theories of probability, fuzzy set, and rough set have been developed. Aiming to provide a common framework for different theories, Pedrycz [46,47] and Bargiela and Pedrycz [48] developed the methods of Granular Computing to bring together the existing plethora of formalisms of these uncertainties in spite of their visibly distinct underpinnings (and ensuing processing). Uncertainty theory, proposed by Liu [19] in 2007, is another tool to handle real life uncertainties and eliminate the problems of small sample size arising out of incomplete information. Up to now, this field has been further developed by numerous researchers and applied to various real-life problems.

In uncertainty theory, an uncertain variable is defined as a function from the uncertainty space to the set of real numbers to characterize the uncertain information. Practically, a series of uncertain variables, which are represented by their corresponding uncertainty distributions, can be used in decision-making systems, such as linear uncertain variables, zigzag

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uncertain variables, normal uncertain variables, and lognormal uncertain variables. With this in mind, many optimization and decision-making problems can be efficiently and effectively handled within the framework of uncertainty theory. For instance, in order to provide a new tool to deal with the shortest path problem with nondeterministic arc lengths, Gao [7] presented the uncertainty distribution of the shortest path length with the aid of uncertainty theory. Gao [8] discussed single facility location problems and constructed two uncertain models, in which vertex demand is non-deterministic and treated as an uncertain variable by using uncertainty theory. Huang and Qiao [14] investigated a multi-period portfolio selection problem under uncertain environment. Han et al. [15] studied the maximum flow problem of uncertain network. Zhang and Peng [44] dealt with uncertain optimal assignment problem with uncertain profits, in which the uncertainty distribution of the optimal assignment profit is given and the α-optimal assignment model is proposed. Some related topics such as uncertain sets, uncertain entropy and uncertain risk analysis have attracted attention of researchers in recent times. For instance, to improve the framework of uncertainty theory, Liu [21] gave a stronger definition of independence of uncertain sets. Yao et al. [38] proposed the concept of sine entropy for uncertain variables to cover the shortcomings of logarithmic entropy. Chen et al. [3] proposed the concept of cross-entropy for uncertain variables and minimum cross-entropy principle.

Practically, however, it is widely recognized that uncertainty often occurs in multi-fold forms in the real life decision systems due to the incompleteness of the prior information. To characterize and handle multi-fold uncertainties, some mathematical tools have been proposed in the literature, including fuzzy random variables [18,34], twofold fuzzy sets [5], type-2 fuzzy sets [45], etc. Type-2 fuzzy sets were introduced by Zadeh [45] to capture the fuzziness of the membership functions in fuzzy set theory. Nowadays, the majority of current researchers have been focusing on studying the algebraic structure of type-2 fuzzy sets and investigating linguistic and numerical uncertainties with the aid of type-2 fuzzy sets, for instance Mendel and John [27], Karnik and Mendel [16] and Nieminen [31]. In addition, Gaxiola et al. [9] presented a new back propagation neural method using type-2 and type-1 fuzzy inference systems. González et al. [10] presented a new edge detection method based on generalization of type-2 fuzzy logic and employed the new approach in the benchmark images and synthetic images to illustrate the advantages of using generalized type-2 fuzzy logic. Melin et al. [28] proposed a novel optimization method in tracking controllers based on type-2 and type-1 fuzzy logic theory to address the tracking problem. Melin and Castillo [30] employed type-2 fuzzy logic in classification and pattern recognition in order to improve results which used type-1 fuzzy logic. Castillo and Melin [4] gave a review about the optimization methods used in the design of type-2 fuzzy systems. Hidalgo et al. [11] proposed an optimization method for designing type-2 fuzzy inference systems based on the size of the footprint of uncertainty by using genetic algorithms. Maldonado et al. [29] gave an application in the field programmable gate array by using the particle swarm optimization of interval type-2 fuzzy systems. Leal-Ramírez et al. [25] presented a new age-structured population growth model to simulate the dynamics of a particular bird species which was evaluated using an interval type-2 fuzzy logic system. In 2007, Liu and Liu [24] developed the theory of type-2 fuzziness from the point of view of credibility axiomatic foundation, and proposed the concepts of fuzzy possibility space and type-2 fuzzy variables. Some applications of credibility-based type-2 fuzzy theory have also been proposed by Qin et al. [35,36], and Chen and Wang [2] among others.

To the best of our knowledge, in uncertainty theory no related study about multi-fold uncertainty has attracted sufficient attention so far, although this phenomenon is common in decision systems. Aiming to provide an effective decision tool to handle multi-fold uncertainty, we are particularly interested in investigating the concepts and basic operations of type-2 uncertain variables. Compared to the existing concepts in the literature, we give Table 1 to specify the detailed characteristics of the different proposed concepts.

It is observed from Table 1 that different types of methods can be employed to handle multi-fold uncertainty. Most significantly, fuzzy random variables were proposed based on probability and fuzzy set theories, as a generalization of random variables, and were used to describe fuzzy random knowledge representation. Both twofold fuzzy sets and type-2 fuzzy sets were proposed within the fuzzy set theory to capture the incompleteness of a priori information. Although the uncertain variables defined by Liu [19] and characterized by uncertainty distributions can be used to handle the single-fold uncertainty, they will be ineffective for the even more complex multi-fold uncertainties. In this sense, defining an efficient mathematical method is a theoretically significant and challenging issue for real-world decision systems. Our research will address this problem explicitly.

Table 1
Characteristics of different methods in handling multi-fold uncertainty.

Research	Type	Characteristics	Definition
Kwakernaak [18]	Fuzzy random variable	Randomness with fuzzy results	Fuzzy-valued random variables
Dubois and Prade [5]	Twofold fuzzy sets	Fuzziness, the memberships of some elements are partly-known	A nested pair of fuzzy sets
Zadeh [45]	Type-2 fuzzy sets	Fuzziness with vague membership function	Membership function takes fuzzy value
Liu [19]	Uncertain variable	Single-fold uncertainty	Described by uncertainty distribution
This paper	Type-2 uncertain variable	Twofold uncertainty	Uncertainty distribution takes uncertain variables