Information Sciences 370–371 (2016) 33–49

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Classification of cellular automata through texture analysis

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ARTICLE INFO

Article history:
Received 9 October 2015
Revised 5 May 2016
Accepted 2 July 2016
Available online 4 July 2016

Keywords:
Cellular automata
Pattern recognition
Image analysis
Texture analysis
Classification

ABSTRACT

The spatio-temporal dynamics of cellular automata (CAs) has attracted the attention of researchers from different fields, mainly mathematics, computer science and engineering, as a consequence of both the intriguing spatio-temporal patterns that these dynamical systems evolve and the fact that they enable the modelling of complex natural phenomena. Yet, to this day, there are only a few studies that focus on the automated classification of cellular automata on the basis of the space-time diagrams they evolve. Here, we present an innovative approach to classify CAs according to Wolfram's classification scheme in an automated way by relying on texture descriptors that capture the nature of the evolved space-time diagrams. More specifically, we propose the use of one of two well-known texture descriptors, namely Local Binary Pattern Variance and Fourier descriptors, to generate features grasping the diagrams' nature, followed by nearest neighbor classification. The performance of this approach is assessed through a cross-validation and by analysing the percentage of pre-classified rules that is required to arrive at an acceptable success rate. The experiments involve the family of elementary CAs and four families of totalistic CAs with neighborhood radii ranging from one to three, and a state space consisting of up to three states. The results show the potential of our proposal with success rates varying between 65% and 98% depending on the size of the training set, which ranges from 10% to 90% of the rules in the CA family at stake. For totalistic CAs, this training set should be classified manually to start the process of automated classification.

1. Introduction

Ever since the dynamics of cellular automata (CAs) could be visualized easily by means of modern computers, the intriguing spatio-temporal patterns they evolve have aroused interest across many disciplines, some of them only loosely connected to the field of computer science. For instance, ecologists and biologists have been relying on CAs for explaining striking spatial patterns in ecosystems [5,23], the growth of tumors [25], as well as the pigmentation of animals [6]. Similarly, chemical systems like the Belousov-Zhabotinsky reaction have been modeled using CAs [17], while this paradigm

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http://dx.doi.org/10.1016/j.ins.2016.07.005
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is also appreciated to evolve realistic cracking patterns [10]. For a more comprehensive overview of CA-based models, the reader is referred to [7,21].

In addition to the use of CAs as models of complex natural phenomena, they have also been investigated from a more theoretical point of view by mathematicians and computer scientists. As CAs constitute the discrete counterparts of partial differential equations, they may be investigated in terms of their dynamical properties, such as equilibrium, stability and sensitive dependence on initial conditions (ICs) [13,18,19,22,26]. Although a detailed overview of the approaches that have been established to characterize CA dynamics is beyond the scope of this paper, it is important to highlight that there are essentially two ways of looking at this problem. The first one traces back to a conjecture by Wolfram [27], which states that all CAs can be classified in one of four behavioral classes based on a visual inspection of the evolved space-time diagrams (STDs). The main drawbacks of this approach lie in the fact that such a visual inspection becomes time consuming if one aims at investigating an entire family of CAs and that the outcome of the classification process might vary as it involves a trace of subjectivity, as such jeopardising its reproducibility. The second approach relies on measures quantifying the dynamics of CAs, such as Lyapunov exponents [23,22], entropies [11], and others [15,29], and are similar in spirit as the ones that are typically used for analyzing other types of dynamical systems. For a comprehensive overview of the measures and approaches that have been developed to classify CAs, we refer the reader to [19].

When it comes to an automated classification of CAs, there are only a few works in literature that have touched upon this issue. Kunkle [14] presents an algorithm based on neural networks for performing this task in the case of elementary CAs and one family of totalistic CAs. This algorithm involves seven parameters, being activity, reverse determinism, sensitivity, absolute activity, neighborhood dominance, activity propagation and incompressibility. In the work of Wuensche [28], CA rules are classified by an entropy-related measure. In both cases, the complexity of the generated patterns is examined over time, which contrasts with our approach that relies on the STD as a whole.

A visual classification of the STDs evolved by CAs according to the classification postulated by Wolfram [27] is related to texture perception by the human visual system. The interaction of texture elements, as well as the statistical order of the gray levels of the texture image, are the main determinants underlying a visual discrimination of textures [4,12]. The pattern properties in texture perception studies serve as an inspiration for our proposal. Its aim is to investigate the potential of texture descriptors in representing the STDs evolved by CAs and to classify them in an automated way according to the classification postulated by Wolfram [27]. In this way, one can overcome the major drawbacks tied up with a manual classification of CAs on the basis of their STDs.

From now on, such STDs will also be referred to as images as they will be envisaged as images and treated using techniques that are well established in image analysis. More precisely, two feature extraction techniques are used, being Local Binary Pattern Variance (LBPV) and Fourier descriptors [8,9]. An extensive experimental study covering five CA families was conducted: the family of so-called elementary CAs (ECAs), as well as four totalistic CA families built upon two or three states and different neighborhoods. In this way, the experimental study includes families with smaller and bigger rule spaces, so families of a relatively less and more complex nature [30]. For each CA family, nearest neighbor classification is used to classify the images, and hence the corresponding rules, based on the extracted feature vectors. Finally, the performance of our approach is assessed in a meaningful way. The results indicate an excellent classification performance for the considered CA families even when using a small training set manually classified to start the process of automated classification. For that reason, this study constitutes a novel contribution to the CA field in that it outlines a powerful method enabling the automated classification of CAs based on a texture analysis of their STDs.

This paper is organized as follows. Section 2 presents the preliminaries, including a CA definition, Wolfram's classification scheme and a brief introduction to the CA families that are at stake throughout this paper. Section 3 describes how the image databases are constructed, as well as the feature descriptors used. The results of the classification experiments are reported in Section 4 together with a discussion of the most important findings, and finally, conclusions are presented in Section 5.

2. Preliminaries

Informally, a CA may be envisaged as an n-dimensional array of cells, each of them bearing one out of a finite number of discrete states, and the states of these cells are updated in discrete time steps according to the states of the cells in their neighborhood [24].

More formally, a CA $\mathscr{C}$ can be represented as a quintuple

$$\mathscr{C} = \langle \mathcal{T}, S, s, N, \Phi \rangle,$$

where

(i) $\mathcal{T}$ is a countably infinite tessellation of the n-dimensional Euclidean space $\mathbb{R}^n$, consisting of cells c_i, $i \in \mathbb{N}$.
(ii) S is a finite set of k states, here $S \subset \mathbb{N}$.
(iii) The output function $s : \mathcal{T} \times \mathbb{N} \to S$ yields the state value of cell c_i at the t-th discrete time step, i.e. $s(c_i, t)$.
(iv) The neighborhood function $N : \mathcal{T} \to \bigcup_{p=1}^{\infty} \mathcal{T}^p$ maps every cell c_i to a finite sequence $N(c_i) = (c_{i_j})_{j=1}^{|N(c_i)|}$, consisting of $|N(c_i)|$ distinct cells c_{i_j}.