A new echo state network with variable memory length

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\textbf{A B S T R A C T}

This paper proposes a new echo state network (ESN) with variable memory length. For input-driven applications, existing ESNs do not fully consider the characteristics of input signals and usually ignore output feedback connections. Therefore, the echo state property of ESN cannot be completely satisfied for a certain period of time, and thus ESNs cannot provide higher accuracy and faster convergence speed for time-series prediction. To overcome the abovementioned problems of existing ESNs, we propose a variable memory length echo state network (VML-ESN) that can adaptively adjust the state update pattern according to the autocorrelation characteristic of the input signals. For different input signals, the reservoir of VML-ESN is composed of different leaky integrator units with multiple delays. Therefore, the reservoir of VML-ESN has a variable state update equation for different types of input signals. A sufficient condition is given to guarantee that the VML-ESN model has the echo state property. The extended Kalman filtering (EKF) method is utilized to obtain the global optimal parameters of VML-ESN. To validate the effectiveness of VML-ESN, we use VML-ESN to predict different types of time series. Simulation results showed that VML-ESN can greatly improve the prediction accuracy and training time for different input signals.

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1. Introduction

In recent years, time-series prediction has been applied in many different fields \cite{14,20,22,27}. To predict a time series, researchers have widely used artificial intelligence methods, such as back-propagation (BP) algorithm \cite{27}, recurrent neural network (RNN) \cite{17,48}, etc. Echo state network (ESN), an improved model of recurrent neural network \cite{19,20}, uses an interconnected recurrent grid of processing neurons called dynamic reservoir to replace the hidden layer of RNN. The advantage of ESN over RNN is that only the output weights need to be trained, while the reservoir weights and input weights usually are given randomly. Thus, compared with RNN, ESN not only can provide a simple and distinctive learning method, but also enables the learning result to obtain higher accuracy \cite{20,23,35}. For purely input-driven applications (for example, time-series prediction \cite{4,13,32,34}, dynamic pattern recognition \cite{36,37,40}, system modeling \cite{31,47}, and filtering or control \cite{15,23,25,41,44}), the output feedback is often ignored to ensure the stability of traditional ESNs. Thus, the current state of a traditional ESN depends only fractionally and indirectly on its previous inputs. This characteristic simply matches the echo

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state property, which implies that the influence of the previous input on the current state will gradually fade to zero, if the time tends to infinity.

More and more researchers are committed to improving the performance of ESN, for instance, improving the training method of ESN [3,4,14,16,22,26,33,45], modifying the state update equation of the reservoir [13,21,29,32,34] and output equation [3], etc. For example, in [22], a robust ESN (RESN) was proposed that uses Bayesian regression to train ESN. In [33], an effective algorithm was proposed to prune reservoir redundant connections by using the correlation of the reservoir states during training. In [4], a novel unsupervised method for designing the reservoir of ESN was proposed. Reference [13] proposed the concept of time-delay reservoir, which considers the signal transmission time between neurons and the finite switch speed of amplifiers for the circuit implementation of a neural network [1,2,7,9,24,39,42,43]. Reference [32] proposed a modular state space of echo state network (MSSESN) in which the state space is divided into several subspaces. Reference [34] proposed an improved ESN model with noise addition (EKF/KF-ESN) in which the additive noises describe the internal state uncertainty and the output uncertainty. In terms of the parameter determination of this prediction model, a nonlinear/linear dual estimation consisting of a nonlinear Kalman filter and a linear one is proposed to perform the supervised learning. The leaky integrator echo state network (Leaky-ESN) proposed in [21] is a modified model of the standard ESN. The reservoir of Leaky-ESN is made from leaky-integrator neurons. Leaky-ESN often ignores its output feedback for purely input-driven applications, and, thus, the current reservoir state of Leaky-ESN can be significantly affected by the current input and the previous input. To illustrate whether Leaky-ESN essentially improves the performance of an ESN, we use Leaky-ESN and the standard ESN to predict a time series, respectively. Here, prediction performance mainly includes the prediction accuracy and the settling time, etc. The settling time denotes a valid training time to establish the network. As we show in the Simulation Experiment section, testing results showed that the prediction performances of Leaky-ESN and the standard ESN have a great difference for different input signals. For example, for a deterministic time series with a very strong correlation of two adjacent moment input signals [21], the prediction performance of Leaky-ESN is better than that of the standard ESN. However, for a stochastic time series with a very weak autocorrelation characteristic [20], the performance of the standard ESN is better than that of Leaky-ESN. For a chaotic time series with a very strong correlation of three adjacent moment input signals [19], the performances of Leaky-ESN and the standard ESN are not very good. We analyze the causes of the phenomena as follows: (1) The standard ESN and Leaky-ESN cannot fully consider the characteristics of the input signal. In fact, different types of input signals usually have different characteristics. The Leaky-ESN model coincides with the autocorrelation of the deterministic time series [21], and, thus, it gives a better prediction performance than the standard ESN. The standard ESN exactly coincides with the autocorrelation of the stochastic time series [20], and, thus, it gives a better prediction performance than Leaky-ESN. However, Leaky-ESN and the standard ESN cannot coincide with the autocorrelation of the chaotic time series [19], and, thus, they should be modified to obtain better prediction performance. Since the echo state property of Leaky-ESN and the standard ESN cannot be completely satisfied for a certain period of time, we have to construct a new ESN that can fully consider the characteristics of the input signals in practical applications. (2) After the stochastic assignment of reservoir weights, the reservoir of the standard ESN is generally fixed and, thus, its spectral radius is not optimal. Leaky-ESN uses the batch gradient descent (BGD) method with constant learning rate to optimize the parameters of the reservoir state update equation, which makes Leaky-ESN exhibit some problems, such as slow convergence speed and longer training time. To solve the abovementioned problems of the standard ESN and Leaky-ESN, this paper proposes a new ESN named variable memory length echo state network (VML-ESN), which can adaptively adjust the state update pattern according to the characteristic of the input signals. For the standard ESN, past inputs implicitly exist in the activation state and then decrease their influence on the activation state of the network. Several studies have addressed the question of how long memory traces can last in such networks, as functions of the network size, connectivity, and input statistics [6,10,11,18,33]. References [6,18] show that the duration of memory traces in any networks cannot exceed the number of neurons (in units of the intrinsic time constant). For VML-ESN, the activation state of VML-ESN is a function of several previous states and current input. This highlights the influence of the input history on the current state. How long can memory traces last in VML-ESN networks? This depends on the characteristic of the network input signals.

The main contributions of this paper are as follows: (1) An improved ESN is proposed, i.e., VML-ESN, which can ensure good prediction performance for different input signals. The reservoir of VML-ESN can express the autocorrelation characteristic of a time series by using a variable memory length. According to a given threshold that is related to the accuracy requirement of a practical application, we can determine the memory length. When the memory length is zero, VML-ESN is turned into the standard ESN. When the memory length is one, VML-ESN is turned into Leaky-ESN. Therefore, the standard ESN and Leaky-ESN are two special cases of VML-ESN. The reservoir units of Leaky-ESN are the same leaky integrator units with a unit delay. However, the reservoir units of VML-ESN are different leaky integrator units with multiple delays or time-varying delays. (2) To improve the convergence rate and the approximation accuracy, we use the EKF method to optimize the parameters of VML-ESN rather than the BGD method. Thus the training process of VML-ESN is converted into a nonlinear filtering problem. (3) To validate VML-ESN, we select different types of time series for the prediction. Compared with the standard ESN and Leaky-ESN, the VML-ESN model can obtain better prediction accuracy and less settling time. Simulation results showed that VML-ESN with a quite small reservoir size can obtain a good prediction performance.

The remaining part of this paper is organized as follows. In Section 2, the theory of the standard ESN and Leaky-ESN is briefly introduced. In Section 3, a new ESN with variable memory ability is proposed. A method that determines the memory length is given. A sufficient condition for VML-ESN is given to guarantee that VML-ESN has the echo state property.