Information Sciences 340–341 (2016) 228–249

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Asymmetric distances to improve *n*-dimensional Pareto fronts graphical analysis

Xavier Blasco[a,*], Gilberto Reynoso-Meza[b], Enrique A. Sánchez Pérez[c], Juan V. Sánchez Pérez[d]

[a] *Instituto Universitario de Automática e Informática Industrial, Universitat Politècnica de València, Camino de Vera S/N, Valencia 46022, Spain*
[b] *Programa de Pós-Graduação em Engenharia de Produção e Sistemas (PPGEPS), Pontificia Universidade Católica do Paraná (PUCPR), Imaculada Conceição 1155, Curitiba 80215-901, Brazil*
[c] *Instituto Universitario de Matemática Pura y Aplicada (IUMPA), Universitat Politècnica de València, Camino de Vera S/N, Valencia 46022, Spain*
[d] *Centro de Tecnologías Físicas: Acústica, Materiales y Astrofísica (CTF:AMA), Universitat Politècnica de València, Camino de Vera S/N, Valencia 46022, Spain*

ARTICLE INFO

Article history:
Received 12 August 2015
Revised 5 November 2015
Accepted 25 December 2015
Available online 7 January 2016

Keywords:
Multi-criteria decision-making
Asymmetric distance
Multi-objective optimization
Decision-making tools
n-dimensional Pareto front
n-dimensional Pareto front visualization

ABSTRACT

Visualization tools and techniques to analyze *n*-dimensional Pareto fronts are valuable for designers and decision makers in order to analyze straightness and drawbacks among design alternatives. Their usefulness is twofold: on the one hand, they provide a practical framework to the decision maker in order to select the preferable solution to be implemented; on the other hand, they may improve the decision maker's design insight,i.e. increasing the designer's knowledge on the multi-objective problem at hand. In this work, an order based asymmetric topology for finite dimensional spaces is introduced. This asymmetric topology, associated to what we called asymmetric distance, provides a theoretical and interpretable framework to analyze design alternatives for *n*-dimensional Pareto fronts. The use of this asymmetric distance will allow a new way to gather dominance and relative distance together. This property can be exploited inside interactive visualization tools. Additionally, a composed norm based on asymmetric distance has been developed. The composed norm allows a fast visualization of designer preferences hypercubes when Level Diagram visualization is used for multidimensional Pareto front analysis. All these proposals are evaluated and validated through different engineering benchmarks; the presented results show the usefulness of this asymmetric topology to improve visualization interpretability.

1. Introduction

It is common to state a design problem as an optimization statement, where a specific cost index must be optimized. However, many real world problems require the fulfillment of a set of requirements and specifications. In that

* Corresponding author. Tel.: +34 686888302; fax: +34 963879579.
E-mail addresses: xblasco@isa.upv.es (X. Blasco), g.reynosomeza@pucpr.br (G. Reynoso-Meza), easancpe@mat.upv.es (E.A. Sánchez Pérez), jusanc@fis.upv.es (J.V. Sánchez Pérez).

http://dx.doi.org/10.1016/j.ins.2015.12.039

case, it is said to have a multi-objective problem (MOP) instead of a single-objective problem. In such statements, it is usual to find that some objectives are in conflict with each other, and therefore a trade-off solution must be found (or selected).

Multi-objective optimization (MOO) can handle these issues in a simple manner, due to its simultaneous optimization approach. In MOO, all the objectives are significant to the designer, and as a consequence, each is optimized. In general, there is no a single solution because no solution is better than the others in all the objectives. Therefore, a set of solutions, the Pareto set $\boldsymbol{\Theta_P}$, is defined and its objective vector set is the Pareto front $\boldsymbol{J_P}$. This set of solutions offers to the decision maker (DM) greater flexibility at the multi-criteria decision-making (MCDM) stage. The role of the designer is to select the best solution according to her/his needs and preferences for a particular situation.

MOO techniques search for a discrete approximation $\boldsymbol{\Theta_P^*}$ of the Pareto set $\boldsymbol{\Theta_P}$ in order to build a useful description $\boldsymbol{J_P^*}$ of the Pareto front that is as good as possible, according to the DM needs. In this way, the DM has a set of solutions for a given problem and a high degree of flexibility when choosing a particular or desired solution. Classic techniques [26] for building this Pareto front have been proposed and multi-objective evolutionary algorithms (MOEAs) have been recently used due to their flexibility when dealing with non-convex and highly constrained functions [8,9].

Once the DM has been provided with a Pareto front approximation $\boldsymbol{J}_P^*$, the DM will need to analyze the trade-off between objectives with two aims: firstly, in order to select the most preferable solution according to her/his preferences: secondly, in order to gain a better design insight of the MOP by *innovization*, that is innovation trough optimization [11].

Several techniques and methods have been developed to facilitate the DM's task [2,12,27,36]. It is widely accepted that visualization tools are valuable and provide to the DM meaningful methods to analyze the Pareto front and take decisions [4,34]. We can recall the desirable characteristics for such visualization techniques noted in [21]: simplicity (it should be easy to understand); persistence (the DM should be able to retain all information in his/her mind); and completeness (all the relevant information should be shown). Moreover, desirable characteristics (at software level) include interactivity with the DM and an intuitive graphical user interface.

For two-dimensional problems (and sometimes for three-dimensional problems) it is usually straightforward to make an accurate graphical analysis of the Pareto front, but the difficulty increases with the dimension of the problem. Common alternatives to tackle an analysis in higher dimensions are Scatter Diagrams, Parallel Coordinates [17,18] and Level Diagrams (LD) [3,29]. Recently, hybrid tools merging Parallel Coordinates, Dendrograms, and Cluster Maps have been proposed [5]. Nevertheless, in spite of the usefulness of those tools in their own merits, new approaches and efforts to improve interpretability and allowing the DM to perform an accurate analysis are valuable.

Recently, asymmetric distances have become a useful and efficient tool for theoreticians and engineers; their intrinsic properties, allowing to state that the distance from *A to B* is not the same as the one from *B to A*, make them a flexible tool to handle problems in a wide variety of domains. They have been successfully used for computer science applications [24], embedding techniques [16], clustering [6,28] and visualizing asymmetric proximity in self organizing maps and multidimensional scaling [22]. Their success is mainly due to their capabilities to incorporate asymmetric relations between data, quite common characteristic in *real world applications* problems. Therefore, incorporating asymmetric distances into visualization techniques, in order to facilitate the DM's analysis of n-dimensional Pareto fronts, could bring an interesting insight, closer to the point of view of the designer improving persistency and completeness.

In this paper, a new order based asymmetric topology is introduced to carry out an analysis between design alternatives for a given MOP. The use of this asymmetric distance (associated to the asymmetric topology) gives a new way to gather dominance and relative distance together with a lower computational cost than the traditional way. This property can be exploited inside interactive visualization tools. Used for coloring purpose it is possible to apply it to several type of visualization, for instance, parallel coordinates, star diagrams, level diagrams, etc. Each visualization technique has its own particular properties. In this article Level Diagrams is used to demonstrate the application of coloring methods based on asymmetric distance. Additionally, a composed norm based on asymmetric distance has been developed. The composed norm allows a fast visualization of designer preferences hypercubes when Level Diagram visualization is used for multidimensional Pareto front analysis. This work is developed on the following assumptions:

- The DM has chosen to tackle a MOP by means of MOO in order to approximate a Pareto front; therefore, this optimization process will provide a set of Pareto optimal solutions, in order to perform a MCDM stage. That is, it is difficult to find the desirable trade-off with other techniques.
- For such MCDM stage, the DM is willing to analyze trade-off among design alternatives, in order to select the most preferable solution according to her/his needs. That is, any (semi)automatic selection procedure will be used.

The remainder of this paper is as follows: in Section 2 some preliminaries and background are stated. In Section 3 the new order based asymmetric topology is defined for n-dimensional Pareto front visualization and the coloring procedures are described. The composed norm for Level Diagram synchronization is presented. In Section 4, this topology is used in several instances, in order to validate its usability for such analysis. Finally, some concluding remarks are given.