



Projective robust nonnegative factorization



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ABSTRACT

Nonnegative matrix factorization (NMF) has been successfully used in many fields as a low-dimensional representation method. Projective nonnegative matrix factorization (PNMF) is a variant of NMF that was proposed to learn a subspace for feature extraction. However, both original NMF and PNMF are sensitive to noise and are unsuitable for feature extraction if data is grossly corrupted. In order to improve the robustness of NMF, a framework named Projective Robust Nonnegative Factorization (PRNF) is proposed in this paper for robust image feature extraction and classification. Since learned projections can weaken noise disturbances, PRNF is more suitable for classification and feature extraction. In order to preserve the geometrical structure of original data, PRNF introduces a graph regularization term which encodes geometrical structure. In the PRNF framework, three algorithms are proposed that add a sparsity constraint on the noise matrix based on $L_{1/2}$ norm, L_1 norm, and $L_{2,1}$ norm, respectively. Robustness and classification performance of the three proposed algorithms are verified with experiments on four face image databases and results are compared with state-of-the-art robust NMF-based algorithms. Experimental results demonstrate the robustness and effectiveness of the algorithms for image classification and feature extraction.

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1. Introduction

Nonnegative matrix factorization (NMF) [5] is a popular matrix factorization technique for dimensionality reduction [15,30] and feature extraction [16,27]. NMF represents original input data as the output of two low-rank nonnegative matrix factors. As a parts-based representation of original data, NMF only employs additive representation. In other words, all elements must be equal to or greater than zero. With non-negativity constraints on the two matrix factors, NMF typically yields a sparse representation of data and has been widely used in pattern recognition [28], computer vision [11,31], and image processing [4,13].

Due to non-negativity constraints, NMF favors to sparse representation, but it does not always result in parts-based representation [19]. To achieve localized NMF representation, Stan et al. [20] proposed local NMF (LNMF), which adds further constraints on the two nonnegative factors. To encode discriminant information into NMF, Wang et al. [32] proposed a novel

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subspace method called Fisher NMF (FNMF), which also produces additive and spatially localized basis images as LNMF. Yuan et al. proposed a novel NMF method that learns spatially localized and parts-based subspace representation of visual patterns [34]. The learned localized features are not only suitable for image compression, but also for object recognition. Yang et al. proposed a novel variant of NMF called projective NMF (PNMF) [35]. Linear and nonlinear extensions of PNMf were introduced in [35].

Other studies try to combine further constraints or discriminant information into NMF to improve performance in classification [17,21], or clustering [6,12]. In [21], the authors proposed discriminant NMF (DNMF), which minimizes within-class scatter and maximizes between-class scatter to learn the nonnegative factorization matrix. Guan et al. [17] encoded manifold regularization and margin maximization into NMF and generated a new method, called manifold regularized discriminative NMF (MD-NMF). In [6], the authors proposed graph regularized NMF (GNMF), which encoded geometrical information of the data space for clustering. Liu et al. [12] proposed a semi-supervised matrix decomposition method, called constrained NMF (CNMF), that encoded label information into NMF for clustering.

Although NMF and its related works have been successfully used in many fields, these methods are sensitive to noise. Therefore, many methods have been proposed that design more robust algorithms to deal with noise. In [7], the authors proposed a robust NMF method (RNMF-21) using the $L_{2,1}$ norm as a loss function that can handle noise and outliers. Zhang et al. [14] proposed a method named robust NMF (RNMF), which assumed that some matrix data entries could be arbitrarily corrupted. Xia et al. [36] proposed a robust kernel NMF using the $L_{2,1}$ norm as a loss function. Shen et al. [3] proposed a robust NMF algorithm (RNMF-1) that jointly approximated a clean data matrix with the product of two nonnegative matrices and estimated the positions and values of outliers or noise.

Recently, robust NMF and its extensions have been successfully applied to various tasks in image processing [18], computer vision [24], and signal processing [13]. However, these algorithms encounter the following practical limitations: 1) in many applications, data contains noise and the proposed robust NMF methods cannot effectively separate them using learning nonnegative low-dimensional representation; thus, learned projections or bases are unsuitable for feature extraction and classification; 2) since samples of data lie on a low dimensional manifold embedded in high-dimensional ambient space, it is necessary to consider the geometrical structure of data to obtain parts-based representation. Existing robust NMF methods do not take this into account. Specifically, existing robust NMF and its extensions do not consider the robustness and manifold structure simultaneously.

Inspired by recent studies in robust NMF, we propose a framework called Projective Robust Nonnegative Factorization (PRNF) to overcome the aforementioned problems. PRNF projects original data on a low-dimensional subspace for robust feature extraction and classification. In the proposed PRNF framework, noise is weakened and the learned projections are more robust to noise, thus PRNF is suitable for classification. To preserve the geometrical structure of original data, PRNF introduces graph regularization to encode the data geometrical structure in the learning steps. In the PRNF framework, we introduce three regularization terms that encode the $L_{1/2}$ norm, L_1 norm, and $L_{2,1}$ norm as a sparsity constraint of the noise matrix, respectively. Experimental results on four public face databases verify robustness and competitive performance against existing robust NMF methods.

The main contributions of this paper are as follows:

- (1) A general framework for robust NMF is proposed to conclude existing robust NMF algorithms.
- (2) A general model of the projective robust NMF algorithm is presented based on the robust NMF model. The proposed framework, referred to as Projective Robust Nonnegative Factorization (PRNF), can not only weaken the influence of noise when learning the optimal projection, but can also maintain the geometrical structure of original data for a better parts-based representation.
- (3) Three algorithms that take different norms as the sparsity constraint on noise data are proposed. We propose three algorithms based on the $L_{1/2}$, L_1 , and $L_{2,1}$ norms for the noise matrix, and analyze their updated rules and prove the algorithms' convergence.

This paper is organized as follows. Section 2 introduces the motivation for the proposed method and framework. Section 3 provides an analysis of different noise distributions. Sections 4, 5, and 6 give details on the three algorithms with the $L_{1/2}$, L_1 , and $L_{2,1}$ norms for the PRNF framework and their theoretical proof of convergence, respectively. Extensive experimental results are presented in Section 7. Section 8 concludes the paper.

2. Framework for projective robust nonnegative factorization

In this section, we introduce the proposed framework, i.e., Projective Robust Nonnegative Factorization (PRNF), for robust face image classification and feature extraction. First, we explain details of the motivation for our method and then present the unified framework with different norms.

2.1. Motivation

Standard NMF is sensitive to outliers and noise [25]. Many algorithms have been proposed to improve the robustness of NMF [3,7,14]. However, existing robust NMF methods have shortcomings as discussed in the Introduction. To the best of our knowledge, there is no algorithm that can separate clean data and noise effectively and maintain the geometrical structure

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