Information Sciences 364–365 (2016) 91–110

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Attribute reduction in ordered decision tables via evidence theory

Wen Sheng Du [a,b], Bao Qing Hu [a,b,*]

[a] School of Mathematics and Statistics, Wuhan University, Wuhan 430072, PR China
[b] Computational Science Hubei Key Laboratory, Wuhan University, Wuhan 430072, PR China

ARTICLE INFO

Article history:
Received 11 December 2014
Revised 6 April 2016
Accepted 12 May 2016
Available online 19 May 2016

Keywords:
Ordered decision table
Attribute reduction
Dominance-based rough set approach
Evidence theory

ABSTRACT

Rough set theory and Dempster–Shafer theory of evidence are two distinct but closely related approaches to modeling and manipulating uncertain information. It is quite natural to set up a hybrid model based on these two theories. In this paper, we investigate the problem of attribute reduction for ordered decision tables based on evidence theory. Belief and plausibility functions, which are strongly connected with lower and upper approximation operators in dominance-based rough set approach, are proposed to define relative belief and plausibility reducts of ordered decision tables. Relationships among various types of relative reducts are thoroughly studied in consistent and inconsistent ordered decision tables. A pair of numeric measures, the inner and outer significance measures of a criterion, is presented to search for a relative belief/plausibility reduct, which is meaningful for practical problems. Some real-world tasks taken from the UCI repository are employed to verify the feasibility and effectiveness of the proposed technique.

1. Introduction

The rough set theory (RST), initiated by Pawlak in the early 1980s [29], serves as an extension of the classical set theory for modeling and processing insufficient and incomplete information in intelligent systems [30]. The starting point of this theory is an observation that objects having the same description are indiscernible or equivalent in view of the available information about them [31]. The lower and upper approximations used to explore knowledge hidden in information systems are then constructed by means of these blocks of indiscernible objects or elementary sets. The main advantage of RST is that it is completely data-driven and no additional information is required. Ever since the inception of RST, it has been successfully applied in many categories such as machine learning, data mining, knowledge discovery in databases, pattern recognition, granular computing and expert systems.

However, the original rough set theory is not able to discover and process inconsistencies coming from consideration of criteria, that is, attributes with preference-ordered domains (scales), such as test scores, product quality, market share, and debt ratio. To address this issue, Greco et al. proposed an extension of RST, which is called the dominance-based rough set approach (DRSA) to take into account the ordering properties of criteria [17,18,43]. This innovation is mainly based on the substitution of the indiscernibility relation by a dominance relation, which permits approximations of unions of ordered classes involved in multi-criteria decision making and multi-criteria sorting problems [6,23,45]. In DRSA, where condition

* Corresponding author at: School of Mathematics and Statistics, Wuhan University, Wuhan 430072, PR China. Tel.: +862768775350.
E-mail addresses: wsdu@whu.edu.cn (W.S. Du), bqhu@whu.edu.cn, bqhujjhe@hotmail.com (B.Q. Hu).

http://dx.doi.org/10.1016/j.ins.2016.05.011
0020-0255/

attributes are criteria and decision classes are preference-ordered, the knowledge to be approximated is a collection of upward and downward unions of decision classes and the dominance cones or granules of knowledge are sets of objects defined by using a dominance relation [10,12,25,37,44,55,56,61].

Another important method used to deal with uncertainty in information systems is the Dempster–Shafer theory of evidence, a generalization of Bayesian theory of subjective judgment. The Dempster–Shafer theory of evidence is also referred to as the Dempster–Shafer theory or just evidence theory. It was originated by Dempster's concept of lower and upper probabilities [8], and extended by Shafer [36] as a theory. The aim is to obtain degrees of belief for one question from subjective probabilities for another related question. The basic representational structure in this theory is a belief structure, which consists of a family of subsets, called focal elements, with associated individual positive weights summing to one. The fundamental numeric measures derived from the belief structure are a dual pair of belief and plausibility functions, which can be employed to measure the lower and upper bounds of the probability.

There are strong connections between rough set theory and Dempster–Shafer theory of evidence [38,39,58]. Belief and plausibility functions which approximate the values of the rough membership assignment were used to define a quality measure over a partition of the universe [40]. Furthermore, it was shown that a generalized decision measure based on generalized decision functions is related to belief and plausibility functions in [41,42]. The lower and upper approximations of a set characterize the non-numeric aspect of the set expressed by the available information whereas the belief and plausibility measures of the same set reflect the numeric aspect of uncertain knowledge. It has been demonstrated that various belief structures are associated with various approximation spaces such that the different dual pairs of lower and upper approximation operators induced by approximation spaces may be used to interpret the corresponding dual pairs of belief and plausibility functions induced by belief structures [7,46,48–52,54,63]. Hence, attribute reduction within RST can be characterized by belief and plausibility functions, which provide an alternative approach to attribute reduction besides the discernibility matrix method and the reduct construction technique.

Based on this fact, many scholars in the rough set community have analyzed the knowledge acquisition in information systems with nominal attributes by employing the evidence theory. For example, Lingras and Yao [27] employed two different generalizations of rough set models to generate plausibility rules with incomplete databases instead of probabilistic rules generated by a Pawlak's rough set model with complete decision tables. Zhang et al. [63] put forward the concepts of belief reduct and plausibility reduct in complete information systems without decisions. Wu et al. [48] discussed knowledge reduction in random information systems via the Dempster–Shafer theory of evidence. Wu [49] investigated attribute reduction in incomplete decision systems based on evidence theory. Wu [52] further used belief and plausibility functions to discuss attribute reduction in random incomplete decision tables.

The evidence theory can also be explicitly employed for attribute reduction in ordered information systems, that is, information systems with ordinal (i.e., ordered) attributes. Nevertheless, little attention has been spent on this problem. Within the authors' knowledge, only partial work was done by Xu et al. [54]. They studied knowledge reduction in ordered information systems based on evidence theory. The belief and plausibility reducts were proposed and their relationships with the classical reduct were examined. Despite this, they did not investigate attribute reduction in ordered decision tables within the framework of evidence theory, which is the main work of our study. The concepts of relative belief reduct and relative plausibility reduct are introduced and a practical approach is established to find a relative belief/plausibility reduct on the basis of numeric characteristics of candidate criteria.

To facilitate our discussion, we give some preliminaries on dominance-based rough sets and Dempster–Shafer theory of evidence in Section 2. In Section 3, attribute reduction in ordered information systems via evidence theory are in discussion, while, in Section 4, the concepts of relative belief and plausibility reducts in ordered decision tables are proposed and the relationships among the new proposed and some existing relative reducts are investigated. Moreover, in Section 3, based on the inner and outer significance measures, a general framework is developed to find a reduct of some type we are concerned with, which is applicable for cases in Section 4 as well. In Section 5, some numerical experiments are conducted to test the validity and efficiency of the proposed method. Section 6 concludes with a summary of the present work and a suggestion for further research.

2. Preliminaries

In this section we mainly review several basic concepts for later use about DRSA in ordered information systems and evidence theory.

2.1. Ordered information systems

An information system is a quadruple $S = (U, AT, V, f)$, where U is a finite nonempty set of objects, AT is a finite nonempty set of attributes, $V = \prod_{a \in AT} V_a$ and V_a is the domain of attribute a, and f: $U \times AT \to V$ is a total function such that $f(x, a) \in V_a$ for every $a \in AT$, $x \in U$, called an information function.

In practical decision-making analysis, we always consider a binary dominance relation between objects that are possibly dominant in terms of values of an attribute set in an information system. In general, an increasing preference and a decreasing preference are considered by a decision maker. If the domain of an attribute is ordered according to an increasing or a decreasing preference, then the attribute is a criterion.