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Design of polynomial fuzzy observer–controller with membership functions using unmeasurable premise variables for nonlinear systems

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ABSTRACT

In this paper, the stability of polynomial fuzzy-model-based (PFMB) observer-control system is investigated via Lyapunov stability theory. The polynomial fuzzy observer with unmeasurable premise variables is designed to estimate the system states. Then the estimated system states are used for the state-feedback control of nonlinear systems. Although the consideration of the polynomial fuzzy model and unmeasurable premise variables enhances the applicability of the fuzzy-model-based (FMB) control strategy, it leads to non-convex stability conditions. Therefore, the refined completing square approach is proposed to derive convex stability conditions in the form of sum of squares (SOS) with less manually designed parameters. In addition, the membership functions of the polynomial observer-controller are optimized by the improved gradient descent method, which outperforms the widely applied parallel distributed compensation (PDC) approach according to a general performance index. Simulation examples are provided to verify the proposed design and optimization scheme.

1. Introduction

Stability analysis and control synthesis for nonlinear systems are difficult to be systematically conducted. Polynomial fuzzy model [49,51] is one of the effective tools to model and analyze nonlinear systems, which is a generalization of Takagi–Sugeno (T–S) fuzzy model [44,45] in terms of modeling capability. Both of them are employed in fuzzy-model-based (FMB) control strategies, which means that the stability analysis and control synthesis are carried out based on the fuzzy model instead of the nonlinear system [14]. Several techniques are widely employed under the FMB control scheme. First, the sector nonlinearity technique [39,50] is exploited to represent the nonlinear system with the fuzzy model. Second, the Lyapunov stability theory [53] is applied to provide sufficient stability conditions. Third, linear matrix inequality (LMI) [46,50] and sum of squares (SOS) approaches [36] are used to describe the stability conditions for the T–S fuzzy model and the polynomial fuzzy model, respectively, which can be solved by convex programming techniques. The SOS conditions can be converted into semidefinite programming problem by SOSTOOLS [35] and then solved by SeDuMi [41]. Furthermore, the parallel distributed compensation (PDC) [53] is implemented for the control synthesis. The feasibility of applying FMB

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control scheme, especially the polynomial fuzzy model and SOS approach, has been demonstrated by existing literature [9,34,47].

With respect to the development of FMB control strategy, the first task is to reduce the conservativeness of stability conditions. Three types of methods are investigated to deal with three sources of conservativeness, respectively. For the source of double fuzzy summation, Pólya's theory [27,37] is exploited to offer progressively necessary and sufficient conditions which generalizes some earlier works [10,26]. For the source of quadratic Lyapunov function, more general types of Lyapunov function candidates such as fuzzy Lyapunov function [5,18,24,29], piecewise linear Lyapunov function [11,12], switching Lyapunov function [21,32] and polynomial Lyapunov function [4,21] have been investigated which include the quadratic one as a special case. For the source of membership-function-independent stability conditions, the membership-function-dependent approach is applied to make the stability conditions depend on membership functions such as using approximated membership functions [17,30], polynomial constraints [38], symbolic variables [22,23,39] and other techniques [3,7,16,18,20].

Another task of the development of FMB control strategy is to extend it to solve control problems [8,15,25,33,40,42,43,54,55]. The T–S fuzzy observer [46] has been extensively investigated to estimate the system states when the system states are not measurable. Considering the case that the premise variables of membership functions are measurable, one can easily apply the separation principle [57] to design the fuzzy observer separately from the fuzzy controller. However, in the case of unmeasurable premise variables, a two-step procedure [31] was required due to the non-convex stability conditions. Since then, several approaches have been proposed to achieve one-step design for unmeasurable premise variables, for example, completing squares [13], matrix decoupling [52], descriptor [6] and Finsler's lemma [1]. While the T–S fuzzy observer is widely studied, the polynomial fuzzy observer receives relatively less attention. The polynomial fuzzy observer was proposed in [48] which generalizes the T–S fuzzy observer. The polynomial system matrices and polynomial input matrices are allowed to exist in the polynomial fuzzy observer, and the observer gains can also be polynomial. Nonetheless, the polynomial fuzzy observer-controller is designed by two steps. The polynomial controller gains have to be obtained first by assuming all system states are measurable. After that, the polynomial observer gains can be subsequently determined. Moreover, only measurable premise variables and constant output matrices are considered, which narrow the applicability. To the best of our knowledge, the polynomial fuzzy observer–controller with one-step design, unmeasurable premise variables and polynomial output matrices has not been investigated.

Under the FMB control strategy, while the PDC approach is mainly employed to design the membership functions for the fuzzy observer–controller, few works have been carried out to optimize the membership functions. Given a performance index (cost function) to evaluate the time response of the system, the membership functions from PDC approach may not be the optimal membership functions to offer the best time response. In [2], the optimal membership functions were designed under the frequency domain such that a desired closed-loop behavior is guaranteed throughout the entire operating domain. However, in some cases, only approximate optimal membership functions can be obtained. In [28], a systematic method for designing optimal membership functions was proposed in a general setting. The variational method is employed to acquire the gradient of the cost function with respect to design parameters in the membership functions, and the gradient descent approach is used to obtain the stationary point of the cost function. Nevertheless, the cost function does not take the control input into account, and the summation-one property of the membership functions is not considered resulting in imprecise calculation of the dynamics of the closed-loop system and the gradients. These limitations of the existing methods motivate us to investigate the optimization of membership functions for the fuzzy observer-controller.

In this paper, we aim to enhance the applicability of FMB control scheme by considering the polynomial fuzzy-model-based (PFMB) observer-controller. Compared with [48], we obtain the polynomial observer gains and controller gains in one step rather than two steps. The premise variables are unmeasurable which are more general than measurable premise variables, and the output matrices are allowed to be polynomial matrices instead of constant matrices. To achieve the one-step design, the completing square approach refining the one in [13] is employed to derive the convex stability conditions in terms of SOS. Compared with [13], the number of manually designed parameters is reduced from 4 to 3, and the polynomial fuzzy model considered in this paper is more general than the T–S fuzzy model. Moreover, we aim to improve the performance of the PFMB observer-control system by optimizing the membership functions of the polynomial fuzzy observer-controller. The optimal membership functions in this paper are understood in the following way: given a cost function, a set of linear (or polynomial) observer–controllers, and the form of membership function with some parameters to be optimized, the optimal membership functions are the ones that combine the linear observer–controllers to form a fuzzy observer–controller which provides the lowest cost subject to the system stability. The gradient descent approach improving the one in [28] is exploited to achieve the optimization, which provides better performance than PDC approach. Compared with [28], the observer-based system is considered in this paper and the cost function is generalized by taking into account the control input. More precise gradients are obtained by considering the summation-one property of the membership functions.

This paper is organized as follows. Some notations and the formulation of polynomial fuzzy model, polynomial fuzzy observer and polynomial fuzzy controller are presented in Section 2. Stability analysis of the PFMB observer-control system is conducted in Section 3. The optimization of membership functions of the polynomial observer–controller is carried out in Section 4. Simulation examples demonstrate the proposed design and optimization method in Section 5. Finally, a conclusion is drawn in Section 6.