Information Sciences 355–356 (2016) 249–264

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

A model based on subjective linguistic preference relations for group decision making problems

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ARTICLE INFO

Article history:
Received 22 June 2015
Revised 14 January 2016
Accepted 26 March 2016
Available online 1 April 2016

Keywords:
Subjective linguistic preference relation
Subjective evaluation
Discrete fuzzy number
Group decision making

ABSTRACT

In a group decision making problem, experts often express their opinions through the so-called preference relations. In recent years, several different definitions of preference relations depending on the framework and the nature of the problem have been introduced. These approaches vary from interval-valued fuzzy preference relations to incomplete fuzzy linguistic preference relations. In this paper, a novel definition of preference relation, the so-called subjective linguistic preference relation, is proposed. These preference relations are based on the concept of subjective evaluations, introduced in the linguistic computational model based on discrete fuzzy numbers. In this framework, the experts have more flexibility to express their opinions and the solid mathematical background of this model is a guarantee of no loss of information. Finally, an example of a multi-expert decision making problem with a hierarchical multi-granular linguistic context is analyzed to illustrate the potential of the proposed method and its advantages with respect to other methods.

1. Introduction

A group decision making (GDM) problem [20] may be defined as a decision problem with several alternatives and experts that try to achieve a common solution taking into account all their opinions. Although data-based approaches are gaining ground in industrial decision processes (see [32,33]), the model-based approaches where experts' evaluations are indispensable to reach the final decision are still dominant to solve decision making problems. The experts' opinions are often based on the use of preferences expressed by the experts, usually through the so-called preference relations. Depending on the nature or complexity of the problem, these preference relations can be expressed in different ways, many times embedded in a fuzzy environment (see for instance [15,36]). Interval-valued fuzzy preference relations, linguistic interval fuzzy preference relations or incomplete fuzzy linguistic preference relations are interesting examples of preference relations among many others [2,11,12,15,20,34]. Thereby, for each problem, the most suitable class of fuzzy preference relations should be chosen. This fact is one of the main reasons why a great number of experts have investigated new families of fuzzy preference rela-

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http://dx.doi.org/10.1016/j.ins.2016.03.040
0020-0255/

tions in recent years to dispose of a great bunch of choices in order to pick out the fuzzy preference relation which better fits the nature of the problem and the experts' opinions.

On the other hand, with the aim of providing more flexibility to the experts' opinions, in [19] a linguistic computational model based on discrete fuzzy numbers whose support is a subset of consecutive natural numbers was proposed. This model uses the concept of subjective evaluations (see [5,6,16,19,25,26]) which provides a greater flexibility to the experts to present their opinions on the alternatives. Subjective evaluations are a kind of discrete fuzzy numbers whose support is a subinterval of the finite chain $L_n = \{0, 1, \ldots, n\}$. This class of fuzzy subsets can be interpreted as a flexibilization of linguistic expressions such as *better than High, between Medium and Very High* or even more complex expressions. In fact, in [16,24], it is proved that the linguistic model based on hesitant fuzzy linguistic term sets (see [27,35]) can be understood as a particular case of the model based on subjective evaluations. Consequently, subjective evaluations have already been used successfully in decision making problems [6,19,25,26]. However, until now, all the research carried out on this model has relied on experts' opinions on each of the alternatives of the problem. Thus, in this paper, a new class of fuzzy preference relations based on subjective evaluations is proposed and some properties of the fuzzy preference relations are studied.

To illustrate the applicability of the model presented in this paper, a medical GDM problem is considered and solved by using subjective linguistic preference relations. The modeling of medical decision making has been among the leading research objectives for decades [17,31]. From the early work in 1979 by Sanchez [29] who introduced the concept of *medical knowledge* studying relationships between symptoms and diseases by means of fuzzy relations until nowadays, many authors have proposed different approaches [4,9,17] such as fuzzy cognitive maps, fuzzy soft sets, intuitionistic fuzzy sets, among many others in medical diagnoses.

This paper is structured as follows. In Section 2, we make a brief review of discrete fuzzy numbers, while in Section 3, the computational linguistic model based on subjective evaluations is recalled. In Section 4, the concept of subjective linguistic preference relation is introduced, its properties studied and the model for GDM problems based on these preference relations is fully described. In Section 5, we present an example of a medical multi-expert decision-making problem in a hierarchical multi-granular linguistic context to illustrate the applicability of the proposed method and its advantages. Finally, in Section 6, a comparison of the linguistic model based on subjective evaluations with some well-known group decision making models is carried out. The paper ends with the conclusions and future work we want to develop.

2. Preliminaries

In this section we will present the main concepts related to discrete fuzzy numbers that will be used later.

By a fuzzy subset of $\mathbb{R}$, we mean a function $A : \mathbb{R} \to [0, 1]$. For each fuzzy subset A, let $A^\alpha = \{x \in \mathbb{R} : A(x) \geq \alpha\}$ for any $\alpha \in (0, 1]$ be its α-level set (or α-cut). By $supp(A)$, we mean the support of A, i.e. the set $\{x \in \mathbb{R} : A(x) > 0\}$.

Let us recall the definition of discrete fuzzy number.

Definition 1 [37]**.** A fuzzy subset A of $\mathbb{R}$ with membership mapping $A : \mathbb{R} \to [0, 1]$ is called a *discrete fuzzy number*, or *dfn* for short, if its support is finite, i.e., there exist $x_1, \ldots, x_n \in \mathbb{R}$ with $x_1 < x_2 < \cdots < x_n$ such that $supp(A) = \{x_1, \ldots, x_n\}$, and there are natural numbers s, t with $1 \leq s \leq t \leq n$ such that:

1. $A(x_i)=1$ for all i with $s \leq i \leq t$. (*core*)
2. $A(x_i) \leq A(x_j)$ for all i, j with $1 \leq i \leq j \leq s$.
3. $A(x_i) \geq A(x_j)$ for all i, j with $t \leq i \leq j \leq n$.

In Fig. 1, a graphical representation of a general discrete fuzzy number is displayed. Note that the images increase till the core, where they are equal to 1, and then they decrease.

From now on, we will denote by L_n the finite chain $L_n = \{0, 1, \ldots, n\}$, by $\mathcal{D}_{L_n}$ the set of all discrete fuzzy numbers whose support is contained in L_n and by $\mathcal{A}_1^{L_n}$ the set of discrete fuzzy numbers whose support is a subinterval of the finite chain L_n.

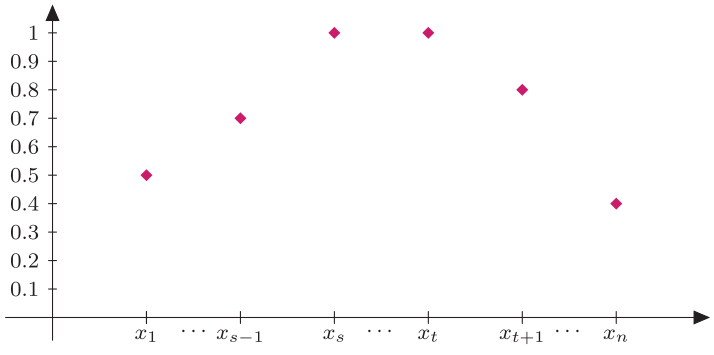

Fig. 1. Graphical representation of a general discrete fuzzy number with support $\{x_1, \ldots, x_n\}$ and core $\{x_s, \ldots, x_t\}$.