Information Sciences 345 (2016) 9–26

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

# Credibilistic mean-entropy models for multi-period portfolio selection with multi-choice aspiration levels

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ARTICLE INFO

*Article history:*
Received 3 September 2015
Revised 26 November 2015
Accepted 20 January 2016
Available online 29 January 2016

*Keywords:*
Fuzzy multi-period portfolio selection
Credibility theory
Fuzzy entropy
Fuzzy mathematical programming
Multi-choice goal programming

ABSTRACT

This paper deals with fuzzy multi-objective multi-period portfolio selection problems. The major criteria used for portfolio selection and rebalancing are wealth, risk, transaction cost, liquidity, and number of assets held in the portfolio. Considering that entropy as a measure of risk does not rely on any particular type of symmetric membership functions of the asset returns and can be computed from the nonmetric data as well, the portfolio risk is quantified using credibilistic entropy of the fuzzy returns. Furthermore, we use multi-choice aspiration levels for the financial goals to better suit the human perceptions such as “the more the better” or “the less the better” regarding return and risk. Two credibility-based fuzzy optimization models are developed using both the discrete choices and interval ranges for the multi-choice aspiration levels of each financial goal. To obtain efficient investment strategies for the entire investment horizon, we use a fuzzy credibilistic programming approach with multi-choice goal programming embedded in it. A real-world empirical application with data-set from an Indian stock market is presented to demonstrate usefulness of the proposed models and the solution approach for multi-period portfolio selection problem in fuzzy environment.



## 1. Introduction

The portfolio selection problem is about forming a portfolio by allocating investor’s wealth among various assets that meets investor-preferences regarding return and risk. Modern portfolio theory is based on mean-variance model [31] in which return is quantified as the mean and risk as the variance. Since, then authors have shown great interest in this field and many efforts have been made to develop portfolio optimization models that better fit to the real-world applications. Konno and Yamazaki [20] used absolute deviation and Speranza [36] used semi-absolute deviation to measure risk in the optimization models for portfolio selection. Yu et al. [39] developed models for diversified portfolios incorporating different entropy measures to quantify the portfolio risk.

Note that the traditional portfolio optimization models for single period investment decision-making provide an one-off decision at the beginning of the investment period and suggests to hold it until the end of the investment period. In practice, however, the investors evaluate and reallocate their wealth from time to time with reference to the changing market conditions. This necessitates the investigation of the multi-period portfolio optimization problem, which has been studied by many researchers using different approaches. Li and Ng [21] proposed dynamic programming approach to deal

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http://dx.doi.org/10.1016/j.ins.2016.01.042
0020-0255/

with the continuous-time setting multi-period portfolio selection problem. Zhu et al. [48] developed generalized mean-variance multi-period portfolio optimization model incorporating bankruptcy. Chen [7] developed an optimization model for multi-period consumption and investment problem using CVaR as the measure of risk. Wei and Ye [38] developed a multi-period mean-variance model for portfolio selection under bankruptcy risk control. Geyer et al. [13] used stochastic linear programming approach to handle the multi-period investment problem. Chen and Song [9] developed a multi-period portfolio selection model based on the multi-factor model in stochastic markets with bankruptcy risk control. Fu et al. [12] discussed a portfolio optimization problem in the continuous-time regime-switching market.

The above referred studies are based on the probabilistic framework where the optimization models were developed under the assumption that the future performance of the assets may be correctly captured by asset data (performance) in the past. That is, it is possible to characterize an asset performance using a random variable with a probability distribution of returns. However, there are many non-random factors, e.g., vagueness and ambiguity associated in a natural way with various types of linguistic expressions such as ‘high risk’, ‘low profit’, and ‘low liquidity’ used by the investors and the investment experts [14,32,33] for the information from the financial markets. Because of vague and ambiguous information, the researchers have used fuzzy set theory [40] in the portfolio selection problem to capture investor-preferences that are subjective in nature. Realizing the importance of fuzzy set theory, more and more researchers have investigated portfolio optimization problem in fuzzy environment, e.g., Barak et al. [1], Bhattacharyya et al. [3], Chen [10], Gupta et al. [15], Gupta et al. [16], Huang [17–19], Li et al. [22], Liu and Zhang [27], Zhang et al. [43].

When compared with the vast literature on fuzzy programming approaches to treat single period portfolio selection problem, there exists few studies focused on fuzzy multi-period portfolio selection problem. Bertsimas and Pachamanova [2] proposed robust optimization formulations of the multi-period portfolio optimization problem that are linear and computationally efficient. Sadjadi et al. [35] proposed a fuzzy multi-period portfolio selection model considering different rates for borrowing and lending. Liu et al. [28] proposed multi-period possibilistic portfolio optimization models using multiple criteria. Zhang et al. [44] proposed a possibilistic mean semivariance-entropy model for fuzzy multi-period portfolio selection. Liu et al. [29] proposed a fuzzy multi-period portfolio optimization model using interval analysis. Zhang et al. [45] formulated multi-period possibilistic mean-variance models. Wang and Liu [37] studied multi-period mean-variance portfolio optimization problem with fixed and proportional transaction costs. Zhang et al. [46] developed a fuzzy programming approach for multi-period portfolio optimization considering return demand and risk control. Zhang and Liu [47] proposed a credibilistic mean-variance model for multi-period portfolio selection problem with risk control. Liu and Zhang [30] formulated a multi-period fuzzy mean-semivariance portfolio optimization model with minimum transaction lots based on possibility theory.

### *1.1. Research issues and need for this paper*

The studies reviewed thus far on multi-period portfolio optimization suffer from a common limitation that a single aspiration level is used for each objective function/goal. In real-world investment decisions, the investor is more interested and comfortable in specifying the multiple aspiration levels for various objective functions/goals. This will help in obtaining a trade-off solution that meets one of stated aspiration levels as closely as possible for each goal. Therefore, it is logical to extend the literature on multi-period portfolio selection using multi-choice aspiration levels to better address the real-world investment problems. Multi-choice goal programming method has been of great interest among the researchers in the recent past and is used in many important real-world applications, e.g., house selection [5], supplier selection [6,24], forest management [8], assignment problem [34]. Despite its growing acceptability, it has not been used yet in multi-period portfolio selection to the best of our knowledge. In our view, the portfolio optimization models can be substantially improved by incorporating multi-choice aspiration levels for each goal, i.e., one goal mapping multiple aspiration levels. This is so because, by allowing the investor to set multi-choice aspiration levels, the underestimation in the achievement of various goals can be avoided in the investment decision-making. For example, the multi-choice aspiration levels for the investment goals help in avoiding the possibility of neglecting assets that may be found suitable but have been omitted from the portfolio to meet the strict single aspiration levels.

Another issue that the review of the studies has revealed is that there is a heavy reliance on the fuzzy variance as a risk measure in the existing optimization models for the multi-period portfolio selection problem. Note that for the fuzzy variance to be an efficient risk measure, the membership function of the asset returns must be symmetrical, which results in limited acceptance of the variance as a risk measure in general. On the other hand, the entropy as a measure of risk does not rely on any particular type of symmetric membership functions of the asset returns and can be computed from the nonmetric data. Hence, it is more practical and reliable to use entropy as a measure of risk for the real-world investment in financial markets. The smaller the entropy value is, the safer the portfolio would be because of less uncertainty. So far, the consideration of entropy as a measure of risk in uncertain environment has not been studied much despite that it is more dynamic and general than the variance and also free from reliance on any specific distribution. Thus, it is practically useful to consider entropy as a measure of risk to obtain investment strategies for the fuzzy multi-period investment decisions.

The need for the present work thus distinctly arises from the utility of fuzzy entropy as a risk measure and multi-choice aspiration levels for the investment goals to obtain investment strategies for the fuzzy multi-period investment decisions.