Information Sciences 345 (2016) 46–64

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Three-way group decisions with decision-theoretic rough sets

Decui Liang[a,*], Dun Liu[b], Agbodah Kobina[a]

[a] School of Management and Economics, University of Electronic Science and Technology of China, Chengdu 610054, China
[b] School of Economics and Management, Southwest Jiaotong University, Chengdu 610031, China

ARTICLE INFO

Article history:
Received 19 May 2015
Revised 15 November 2015
Accepted 11 January 2016
Available online 2 February 2016

Keywords:
Three-way decisions
Decision-theoretic rough sets
Group decision making
Granular computing
Interval analysis

ABSTRACT

On consideration of the effectiveness of group decision making (GDM) in practical complex decision problems, we introduce GDM into three-way decisions with decision-theoretic rough sets (DTRSs) and propose GDM-based three-way decisions. GDM-based three-way decisions extend the range of applications of three-way decisions with DTRSs and provide a novel interpretation of the determination of loss functions. Based on DTRSs, we firstly focus on analyzing the determination for the loss functions under the GDM environment. With the aid of the principle of justifiable granularity, we adopt the important and majority suggestions of experts to measure each loss function, which supports a coherent way of designing information granules in presence of numerics. In this case, the loss functions are determined in the form of interval-valued information granule. By using the interval comparison method, we further deduce the three-way decisions and design a corresponding decision procedure of GDM-based three-way decisions. Then, an example of strategy supply selection is given to elaborate the GDM-based three-way decisions. Finally, we validate the performance of our proposed method by experimental analysis.

1. Introduction

Three-way decisions are commonly encountered in problem solving strategies [14,48,50]. As a novel method, it has been applied in many fields, such as ranking and resource allocation [22], investment decisions [20,27], information filtering [16], text classification [17], risk decisions [18,22], cluster analysis [25,52,54], government decisions [29], web-based support systems [45,47], recommender systems [56], email spam filtering [59], etc. The decisions of acceptance, rejection and non-commitment are the general interpretations of three-way decisions. On the basis of different decision problems, we need to explain the associated semantics of three-way decisions, respectively. For instance, in medical decision making, they may be interpreted as treatment, no treatment and further testing. The idea of three-way decisions is consistent with human's cognitions to solve the problem in the real world. With respect to the research content of three-way decisions, decision-theoretic rough sets (DTRSs) are typical models of three-way decisions [7,19,31,33,48,50,53,57,58]. DTRSs were proposed by Yao et al. [48,49] in the framework of the Bayesian decision procedure [8] and rough set theory [34]. It not only considers decision risk factors, but also involves three types of decision actions, i.e., the acceptance, the non-commitment and the rejection [28,32,40]. Three-way decisions with DTRSs are derived from the minimum of the risk of decision making. As an extension model of rough sets [34], DTRSs consider the practical semantics and vastly push the development of three-way decisions. For example, on basis of the costs of misclassification, Greco et al. [12] proposed a new dominance-based rough

* Corresponding author.
E-mail addresses: decuiliang@126.com (D. Liang), newton83@163.com (D. Liu), kobinaagbodah@yahoo.com (A. Kobina).

http://dx.doi.org/10.1016/j.ins.2016.01.065
0020-0255/

set approach. In the viewpoint of the optimization, Jia et al. [15] discussed the determination of the thresholds and the reduction. Qian et al. [39] developed a multi-granulation DTRSs.

For three-way decisions with DTRSs, the determination of loss function is a pivotal step in the decision procedure [22]. For example, Li and Zhou [18] discussed the determination of loss functions based on different attitudes of decision makers. Lingras et al. [25] redefined a loss function using the rough clusters. By introducing game theory into probabilistic rough sets, Yao and Herbert [46] proposed game-theoretic rough sets (GTRSs) to determine the values of thresholds. With regard to the loss function of DTRSs, Yu et al. [54] constructed a new clustering validity evaluation function. Liu et al. [27] used the profit function to measure the loss function for the investment decision-making. During the calculation procedure, Yao [50] employed relative values to estimate loss functions. Liang et al. [20], Liang and Liu [21–24] and Liu et al. [30] effectively estimated the loss function with the aid of fuzzy numbers and intervals. Considering the preferences of decision makers, Yang and Yao [44] discussed some aggregations of the thresholds. They extend the range of applications of three-way decisions with DTRSs.

With the aforementioned literatures, the determination of loss function of DTRSs is associated with decision makers. The loss functions in the most of existing studies are evaluated by single decision maker. In the face of complicated decision environment, the single decision maker may be difficult to make a reasonable decision [20,43,44], e.g., limited domain knowledge of decision maker, tight deadlines, limited budgets [20]. Fortunately, group decision making (GDM) provides an effective evaluation approach [30], which can aggregate the wisdom of the different domain experts and effectively cope with risk decision problems [20,41,43]. For example, at a plan moment of new production development, the development decision not only depends on the senior management, but also should rely on the opinions of the engineers and salesmen. GDM also exists in our daily life, e.g., the strategic supplier selection [3], the public-private partnership (PPP) project investment [20], etc. However, we may face the inconsistency among experts under the GDM environment [43]. For example, the score of one student is evaluated by four experts. The results are 55, 82, 85 and 90. Obviously, there is a large difference between 55 and 90. Reaching consensus in GDM becomes an essential step [20]. The consensus among experts reflects their consistent opinions. Generally speaking, the combination of GDM and three-way decisions has not received sufficient research attentions. The loss functions of DTRSs evaluated by GDM need to be deeply investigated.

Considering the effectiveness of GDM in practical complex decision problems, we explore novel three-way decisions with DTRSs. Under the GDM environment, we mainly discuss the determination of loss function and deduce three-way decisions. GDM provides a novel semantic interpretation for the loss functions of three-way decisions. Meanwhile, it also may bring the inconsistency among the opinions of multiple experts [20]. Based on a group of numerics, Pedrycz [38] systematically discussed the building of information granules with the aid of the principle of justifiable granularity, which can abstract the meaning of numerics. The principle of justifiable granularity is a new approach to aggregate the group opinion of multiple experts [36,37,42]. It can get the important and majority suggestions of experts and improve the evaluation effectiveness of GDM. The interval format is a typical information granule [21]. Thus, we adopt the principle of justifiable granularity to abstract the experts' evaluation results and determine the loss functions with intervals. By utilizing a suitable interval analysis method, we further derive three-way decisions. Then, the decision procedure of GDM-based three-way decisions is designed. Taking the strategy supply selection as an example, we illustrate the GDM-based three-way decisions. Comparing with the results of Yang and Yao [44], we focus on the determination of loss functions and involve the inconsistency among the opinions of multiple experts. The main contributions of our work can be identified as follows: (1) We extend the range of applications of three-way decisions to GDM and provide a novel interpretation of the determination of loss functions ; (2) The principle of justifiable granularity explains the imprecise origin of the existing literatures, e.g., Liang et al. [20], Liang and Liu [21–23].

The remainder of this paper is organized as follows. Section 2 lists some symbols used in this paper in advance. Section 3 provides basic concepts of three-way decisions with DTRSs. Under the GDM environment, GDM-based three-way decisions are proposed in Section 4. Then, a case study of strategy supply selection is given to illustrate our proposed method in Section 5. In Section 6, we further validate the performance of our proposed method by experimental analysis. Section 7 concludes the paper and outlines the future work.

2. Notations

Under the GDM environment, we extend three-way decisions with DTRSs to a general situation and propose GDM-based three-way decisions. Many symbols are involved in this study and for clarity most of them are summarized in Table 1.

3. A brief review of three-way decisions with decision-theoretic rough sets (DTRSs)

In this section, we briefly review the three-way decisions derived from DTRSs. In light of Bayesian decision procedure [8] and rough set theory [34], Yao [48–50] assumed that DTRSs were composed of two states and three actions. With respect to the relevant concepts of rough sets, the set of states is given by $\Omega = \{C, \neg C\}$ indicating that an object is in C and not in C, respectively. In the context of different decision making problems, we define their own meanings of the states. The set of actions is given by $\mathcal{A} = \{a_P, a_B, a_N\}$, where a_P, a_B, and a_N represent three actions when classifying object x, namely, deciding $x \in \mathrm{POS}(C)$, deciding x should be further investigated $x \in \mathrm{BND}(C)$, and deciding $x \in \mathrm{NEG}(C)$, respectively. At this point, POS(C), BND(C) and NEG(C) comprise three-way decisions, which correspond to the acceptance decision, the