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Capacity factors in a point-to-point network

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ABSTRACT

In this study, we investigate some properties of capacity factors (CFs) that may affect the link failure problem in network coding. Roughly speaking, a CF is an edge set where its deletion will reduce the maximum flow, whereas deleting any proper subset of this CF will not. More generally, a k-CF is a minimal (not minimum) edge set, where its removal will decrease the maximum network flow by at least k. First, under a point-to-point acyclic scenario, we characterize all the edges contained in some CFs and we propose an efficient classification algorithm. We show that all of the edges on some s–t path in a point-to-point acyclic network belong to some 2-CF, and we also present some other properties of CFs. Some computational hardness results related to CFs are obtained. We prove that determining the existence of a CF in a cyclic network with a size greater than a given number is an NP-complete problem and that the time complexity of computing its capacity rank is lower bounded by that of solving its maximal flow. In addition, we propose an analogous definition of CFs on vertices as a special case that captures edge CFs.

1. Introduction

Reliability is a critical theme during topology design for communication networks. In traditional combinatorial network theory, the concept of edge connectivity is employed to evaluate the reliability of a network when edge failures occur [2]. Numerous studies have been focused on the connectivity of a network and related concepts such as super-connectivity and conditional connectivity [2,6,8].

A recent study of network coding [1] showed that coding the internal vertices can obtain the optimal capacity of a multicast network, which is upper bounded by the maximum flow or minimum cut. However, this optimal capacity may not be obtained using a traditional routing scheme.

This raises the following question: is it still appropriate to evaluate the reliability of a network-coding-based network using traditional concepts such as edge connectivity, as mentioned above? For traditional networks where only routing schemes are employed, communication will not be disrupted during edge failures provided that at least one path still exists from the source to the sink vertex. However, the communication will be degraded in network coding-based networks, even if the failure of an edge set reduces the number of disjoint paths between source and sink vertices, because the network capacity is decreased.

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Koetter et al. [10] first provided brief descriptions of edge failure problems in network coding-based networks. Cai and Fan [3] formally proposed the concepts of a capacity factor (CF) and a capacity rank. The capacity rank characterizes the criticality of a link for network communication. If there are no CFs present on an edge, the capacity rank of the edge is defined as ∞. Recently, some related work was presented in [5] regarding k-Route Cut, which is the minimum number of edges that allows the connectivity of every pair of source and sink vertices below k. Chuzhoy et al. developed some approximation algorithms as well as proving some computational hardness results.

An open problem was proposed in [3] about how to compute the capacity rank of a given edge. In the present study, we provide a partial answer to this problem by obtaining an equivalent condition for $\mathrm{CR}(e) = \infty$. Based on this result, it is easy to develop an algorithm in $O(|V|^3)$ to determine whether the capacity rank of a given edge is finite. We cannot find an efficient algorithm to compute the capacity rank of a general network, or prove that the problem is NP-hard, but we obtain some related computational hardness results. For example, we prove that determining the existence of a CF with a size larger than a given number is NP-complete and we show that the time complexity of computing the capacity rank is lower bounded by solving the maximal flow.

Although network coding for a single-source single-sink network is not beneficial, our results are focused mainly on a single-source single-sink scenario for two reasons: the study of point-to-point networks can provide insights into multicast scenarios; and a CF is a natural concept, which might be interesting in its own right.

The remainder of this paper is organized as follows. In section 2, we review some basic definitions, notations, and related results. In section 3, we investigate some properties of CFs, including point-to-point scenarios and multicast scenarios. In section 4, we propose an algorithm for calculating the D-set and H-set, where we prove its correctness and analyze its time complexity. In section 5, we present some computational hardness results related to CFs.

2. Preliminaries

In this section, we review some basic definitions, notations, and results, which will be used in the sequel.

A communication network is a collection of directed links that connect transmitters, switches, and receivers. It is often represented by a 4-tuple $\mathcal{N} = (V, E, S, T)$, where V is the vertex (node) set, E is the edge (link) set, S is the source vertices set, and T is the sink vertices set. A communication network $\mathcal{N}$ is called a point-to-point communication network if $|S| = |T| = 1$, which is denoted by $\mathcal{N} = (V, E, s, t)$, where s is the source vertex and t is the sink vertex.

Without any loss of generality, we may assume that all of the links in a network have unit capacity, i.e., 1 bit per transmission slot. For $u, v \in V$, $\langle u, v\rangle$ is denoted by the edge from u to v. If there are k edges from u to v, $\langle u, v\rangle_k$ denotes the set of edges from u to v, which we simply denote by $\langle u, v\rangle$ provided that there is no ambiguity. For an edge $e = \langle u, v\rangle$, u is called the tail of e, which is denoted by $tail(e)$, and v is called the head of e, which is denoted by $head(e)$.

If $F \subseteq E, \mathcal{N} \setminus F$ denotes the network obtained by deleting edges in F from $\mathcal{N}$. If $V' \subseteq V, \mathcal{N}(V')$ denotes the network of vertices in V' and the edges among V' of $\mathcal{N}$, where the vertex-induced network of $\mathcal{N}$ is called V'. For $V_1, V_2 \subseteq V, [V_1, V_2]$ denotes the set of all links with tails in V_1 and heads in V_2. For a network $\mathcal{N} = (V, E, S, T)$, an S-T cut of $\mathcal{N}$ is $[V_1, \overline{V_1}]$ such that $S \subseteq V_1$ and $T \cap V_1 = \emptyset$. A minimal S–T cut of $\mathcal{N}$ is an S–T cut with a minimal size, which is denoted by $C_{\mathcal{N}}(S, T)$.

It is well known that for a point-to-point network $\mathcal{N} = (V, E, s, t)$, the maximal flow from s to t is equal to the minimal s–t cut of $\mathcal{N}$ and a feasible flow is a maximal flow if and only if there is no augmenting path in the corresponding residual network (*Max-flow Min-cut Theorem* [4,12]). If each link in $\mathcal{N}$ has unit capacity, then the maximal flow of $\mathcal{N}$ corresponds to f edge-disjoint paths from s to t in $\mathcal{N}$ (*Integrality Theorem* [12]).

Let $\mathcal{N} = (V, E, s, t)$ be a point-to-point network. For any vertex $v \in V$, we assume that a path from s to t exists in $\mathcal{N}$, which passes the vertex v. Otherwise, we may delete the vertex v because v is not required for the communication between s and t.

Definition 2.1 [3]. Let $\mathcal{N} = (V, E, s, t)$ be a point-to-point network. A nonempty subset F of E is CF of $\mathcal{N}$ if and only if the following two conditions hold:

1. $C_{\mathcal{N}\setminus F}(s, t) < C_{\mathcal{N}}(s, t)$.
2. $C_{\mathcal{N}\setminus F'} = C_{\mathcal{N}}(s, t)$ for any proper subset F' *of* F.

$\mathcal{N} \setminus F$ denotes the induced network formed by deleting F in $\mathcal{N}$.

By this definition, for a CF F, adding any one edge $e \in F$ in the point-to-point network $\mathcal{N} \setminus F$ will increase the maximal flow. Since adding one edge can increase the maximal flow by at most 1, we have $C_{\mathcal{N}\setminus F}(s, t) = C_{\mathcal{N}}(s, t) - 1$.

In general, we can define a kth order CF (k-CF) as follows, where our motivation will be clear in the multicast scenario.

Definition 2.2. Let $\mathcal{N} = (V, E, s, t)$ be a point-to-point network. A nonempty subset F of E is a kth order CF (k-CF) of $\mathcal{N}$ if and only if the following two conditions hold: