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Edge-fault-tolerant pancyclicity of arrangement graphs ☆

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ABSTRACT

The arrangement graph $A_{n,k}$ is a well-known interconnection network. Day and Tripathi proved that $A_{n,k}$ is pancyclic for $n-k \geqslant 2$. In this paper, we improve this result, and we demonstrate that $A_{n,k}$ is also pancyclic even if it has no more than $(k(n-k)-2)$ faulty edges for $n-k \geqslant 2$. Our result is optimal concerning the edge fault-tolerance.

1. Introduction

The interconnection network is an important research area for parallel and distributed computer systems. Usually, the topology of a network can be represented as a graph in which the vertices represent processors and the edges represent the communication links.

The star graph, which was proposed by Akers et al. [1], is a well-known interconnection network. As a generalization of the star graph, Day and Tripathi [5] proposed the arrangement graph. For a positive integer s, let $\langle s\rangle$ denote the set $\{1,2,\ldots,s\}$. Given two positive integers n and k with $n > k \geqslant 1$, the (n,k)-arrangement graph $A_{n,k}$ is the graph that has the vertex set $V(A_{n,k}) = \{u = u_1u_2\ldots u_k | u_i \in \langle n\rangle, u_i \neq u_j \text{ if } i \neq j\}$ and the edge set $E(A_{n,k}) = \{(p,q) | p,q \in V(A_{n,k})$, and p,q differ in exactly one position$\}$. From the definition, we know that $A_{n,k}$ is a regular graph of degree $k(n-k)$ with $\frac{n!}{(n-k)!}$ vertices, $A_{n,1}$ is isomorphic to the complete graph K_n, and $A_{n,n-1}$ is isomorphic to the n-dimensional star graph. Moreover, $A_{n,k}$ is vertex-transitive and edge-transitive [5]. Fig. 1 shows the arrangement graph $A_{4,2}$.

For $i \in \langle n\rangle, j \in \langle k\rangle$, suppose that $A_{n,k}^{(j:i)}$ denotes the subgraph of $A_{n,k}$ that is induced by $V\left(A_{n,k}^{(j:i)}\right) = \{p\,|\,p = p_1p_2\ldots p_k$ and $p_j = i\}$. Obviously, $\{V(A_{n,k}^{(j:i)})\,|\,1 \leqslant i \leqslant n\}$ forms a partition of $V(A_{n,k})$ and each $A_{n,k}^{(j:i)}$ is isomorphic to $A_{n-1,k-1}$. Then $A_{n,k}$ can be recursively constructed from n copies of $A_{n-1,k-1}$ and every two copies have $\frac{(n-2)!}{(n-k-1)!}$ edges between them. We follow [4] for graph-theoretical terminologies and notations. For two paths $P = \langle u_0,u_1,\ldots,u_m\rangle$ and $P' = \langle u_m,u_{m+1},\ldots,u_n\rangle, P + P'$ denote the path $\langle u_0,u_1,\ldots,u_m,u_{m+1},\ldots,u_n\rangle$.

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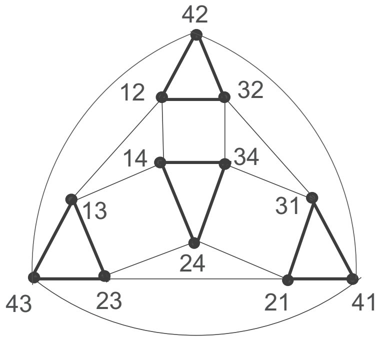

Fig. 1. The arrangement graph $A_{4,2}$.

In interconnection networks, the problem of simulating one network by another is modeled as a graph-embedding problem. There are several reasons why such an embedding is important [24]. For example, the execution of an efficient algorithm requires certain topological structures. Thus, it is desired to provide logically a specific topological structure throughout the execution of the algorithm in the network design.

The cycle embedding problem is one of the most popular embedding problems. This problem is to find a cycle of given length in a graph. A graph G is Hamiltonian if it has a cycle of length $|V(G)|$. If there exists a Hamiltonian path (a path of length $|V(G)|-1$) between any two vertices of G, then the graph G is said to be Hamiltonian connected. A graph G is pancyclic if it has cycles of each length from g to $|V(G)|$ where g is the length of a shortest cycle of G. Let $d_G(u,v)$ denote the length of a shortest path between vertices u and v in G. A graph G is panconnected if for any two vertices x, y in G, there exist paths between x and y of each length from $d_G(x,y)$ to $|V(G)|-1$. The pancyclicity is an important metric when embedding cycles of any length into the topology of a network. A large amount of related work has appeared in the literature [2,3,6,13–15,23].

Because some components in a graph could fail sometimes, it is more practical to study graphs with faults. A graph G is k-fault-tolerant pancyclic (resp. Hamiltonian, Hamiltonian connected, panconnected) if $G-F$ remains pancyclic (resp. Hamiltonian, Hamiltonian connected, panconnected) for $F \subseteq V(G) \cup E(G), |F| \leqslant k$. A graph G is k-edge-fault-tolerant pancyclic (resp. Hamiltonian, Hamiltonian connected, panconnected) if $G-F$ remains pancyclic (resp. Hamiltonian, Hamiltonian connected, panconnected) for $F \subseteq E(G), |F| \leqslant k$. The fault-tolerant pancyclicity has been investigated widely. There is a substantial amount of related literature [7–12,15,16,19–22,25].

For $n-k=1$, the cycle embedding problems of the arrangement graph $A_{n,k}$, which is a star graph, have been discussed in [7,13,15,21,25]. For $n-k \geqslant 2$, Day and Tripathi [6] proved that $A_{n,k}$ is pancyclic. Teng et al. [18] proved that $A_{n,k}$ is panconnected. Concerning fault tolerance, Hsieh et al. [10] studied the existence of Hamiltonian cycles in faulty arrangement graphs. Hsu et al. [12] proved that $A_{n,k}$ is $(k(n-k)-2)$-fault-tolerant Hamiltonian if $n-k \geqslant 2$. Lo and Chen [17] proved that $A_{n,k}$ is $(k(n-k)-2)$-edge-fault-tolerant Hamiltonian connected if all faulty edges are not adjacent to the same vertex. In this paper, we prove that $A_{n,k}$ is $(k(n-k)-2)$-edge-fault-tolerant pancyclic if $n-k \geqslant 2$. If there are $k(n-k)-1$ faulty edges and all of them are adjacent to the same vertex in $A_{n,k}$, then $A_{n,k}-F$ is not Hamiltonian. This finding demonstrates that our result is optimal with respect to edge fault-tolerance.

2. Some properties of the arrangement graphs

First, we give some of the known results about the arrangement graph.

Theorem 1 (Day and Tripathi [6]). *The arrangement graph $A_{n,k}$ is pancyclic for $n-k \geqslant 2$.* □

Theorem 2 (Teng et al. [18]). *The arrangement graph $A_{n,k}$ is panconnected for $n-k \geqslant 2$.* □

Theorem 3 (Hsu et al. [12]). *The arrangement graph $A_{n,k}$ is $(k(n-k)-2)$-fault-tolerant Hamiltonian, and $(k(n-k)-3)$-fault-tolerant Hamiltonian-connected for $n-k \geqslant 2$.* □

Denote $E^{l=i,j} = \left\{(u,v) \in E(A_{n,k}) : u \in V\left(A_{n,k}^{(l:i)}\right), v \in V\left(A_{n,k}^{(l:j)}\right)\right\}$. Then, $|E^{l=i,j}| = \frac{(n-2)!}{(n-k-1)!}$. Specifically, we use $A_{n,k}^{i}$ to denote $A_{n,k}^{(k:i)}$ and $E^{i,j}$ to denote $E^{k=i,j}$ in $A_{n,k}$. For a subset I of $\langle n \rangle$, $A_{n,k}^{I}$ denotes the subgraph of $A_{n,k}$ that is induced by $\bigcup_{i \in I} V(A_{n,k}^{i})$.

Hsu, Li, Tan and Hsu proved the theorem below.