Information Sciences 334–335 (2016) 103–121

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

# A clique-based exact method for optimal winner determination in combinatorial auctions

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ARTICLE INFO

*Article history:*
Received 4 August 2014
Revised 5 October 2015
Accepted 12 November 2015
Available online 2 December 2015

*Keywords:*
Winner determination
Combinatorial auctions
Maximum weight clique
Vertex coloring
Exact search

ABSTRACT

Given a set of items to sell and a set of combinatorial bids, the Winner Determination Problem (WDP) in combinatorial auctions is to determine an allocation of items to bidders such that the auctioneer's revenue is maximized while each item is allocated to at most one bidder. WDP is at the core of numerous relevant applications in multi-agent systems, e-commerce and many others. We develop a clique-based branch-and-bound approach for WDP which relies on a transformation of WDP into the maximum weight clique problem. To ensure the efficiency of the proposed search algorithm, we introduce specific bounding and branching strategies using a dedicated vertex coloring procedure and a specific vertex sorting technique. We assess the performance of the proposed algorithm on a large collection of benchmark instances in comparison with the CPLEX 12.4 solver and other approaches. Computational results show that this clique-based method constitutes a valuable and complementary approach for WDP relative to the existing methods.



## 1. Introduction

Combinatorial auctions (CAs) allow bidders to buy entire bundles of goods (or items) in a single transaction [6]. One key issue in CAs is the winner determination problem (WDP) [17]. Given a set of combinatorial bids, each bid being defined by a subset of items with a price, two bids are conflicting if they share at least one item. WDP is to determine a conflict-free allocation of items to bidders (the auctioneer can keep some of the items) such that the auctioneer's revenue is maximized.

In terms of computational complexity, WDP is known to be NP-hard [25]. From the practical point of view, WDP is at the core of a number of relevant applications like cloud computing [26], electronic commerce [39], intelligent transportation systems [30,39], logistics services [39] and production management [24]. The computational challenge of WDP and its practical relevance have motivated the development of a variety of solution approaches in recent years, including both heuristic and exact methods. We provide a review of the main existing methods in the literature in Section 2.

In this paper, we are interested in solving WDP exactly using a clique-based approach. Indeed, it is known that WDP is equivalent to the weight set packing problem [39], and can be reduced to the maximum weight clique problem (MWCP). The first study on the clique-based approach for WDP was explored very recently in [36] where a heuristic is applied to approximate the transformed MWCP problem. In this work, we explore an exact approach with an effective branch-and-bound algorithm

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http://dx.doi.org/10.1016/j.ins.2015.11.029
0020-0255/

(called *MaxWClique*). To get tight upper bounds on the maximum weight clique, we devise a dedicated vertex coloring heuristic which groups vertices of the largest possible weight into a same color class. In vertex coloring, vertices in a graph are assigned a color such that pairwise adjacent vertices are colored differently. The sum of the weights of the color classes produced in the process is an upper bound to the maximum weight clique in the graph. In addition, to prune the search tree effectively, the algorithm employs a global branching rule by presenting the vertices to the coloring procedure in a non-increasing weight order to obtain tight bounds.

The rest of this paper is organized as follows. In Section 2, we provide a literature review of the most representative approaches for WDP as well as MWCP and summarize the main contributions of our work. In Section 3, we establish the connections between the winner determination problem and the maximum weight clique problem. In Section 4, we present the clique-based branch-and-bound algorithm for MWCP (and WDP). In Section 5, we provide computational results of extensive experiments on three sets of WDP benchmark instances in the literature. In Section 6, we provide some insights on the performance of the proposed approach and discuss the classes of WDP instances most suitable for our clique-based approach. Perspectives and concluding remarks are provided in Sections 7 and 8 respectively.

## 2. Literature review and main contributions

In this section, we provide a literature review on the most representative approaches for WDP and MWCP, followed by a summary of the main contributions of our work.

### 2.1. Literature review on algorithms for the winner determination problem

The computational challenge of WDP and its wide practical applications have motivated a variety of solution approaches in the literature, including both heuristic and exact methods.

Heuristic methods are designed to find approximate solutions within acceptable computing time limits, but without provable optimal guarantee of the attained solutions. These methods are often applied when an optimal solution cannot be achieved or is not required. Some representative heuristic algorithms for WDP include a stochastic local search method (Casanova) [15], a hybrid algorithm combining simulated annealing with branch-and-bound (SAGII) [8], a hybrid genetic algorithm [3], a crossover-based tabu search algorithm [32] and a multi-neighborhood tabu search algorithm [36] which explores the cliquebased approach from a heuristic perspective.

On the other hand, considerable effort has been devoted to developing various exact methods for WDP. Attempts to apply exact methods to solve WDP (under the name of set packing) can be found as early as in the beginning of 1970s [22]. Many other solution methods have appeared in the literature ever since. Most exact algorithms are based on the general branch-and-bound (B&B) framework and branch on bids to find optimal allocations. Representative examples include the combinatorial auction structural search (CASS) [10], the Combinatorial Auction Multi-Unit Search (CAMUS) [18], the BOB algorithm [28], the CABOB algorithm [29], and the linear programming based B&B algorithm [20]. These B&B methods differ from each other mainly by (1) specific techniques to determine the lower and upper bounds, (2) their branching strategies and (3) some other techniques like preprocessing, decomposition of the bid graph, and identifying and solving tractable special cases. Especially, the upper-bounding methods play a key role to the performance of these B&B algorithms, and a typical upper-bounding method uses linear programming relaxations of the set packing formulation [20,29]. In addition, other mathematical formulations for WDP have also been studied within a branch-and-cut algorithm [7], a branch-and-price algorithm [9] and a dynamic programming algorithm [25]. However, these last methods do not seem to perform better than the integer linear programming CPLEX solver using a natural formulation of the problem, which indeed shows an excellent performance in many cases [1,8,29].

### 2.2. Literature review on algorithms for the maximum weight clique problem

Though various exact algorithms have been proposed for the unweighted case of the maximum clique problem (see e.g., [37]), MWCP is somewhat less studied in the literature. Yet, several exact algorithms have been proposed to solve this problem.

The B&B algorithm proposed by Östergård [21] (called *Cliquer*) is among the most popular and influential MWCP algorithms. *Cliquer* relies on an iterative deepening strategy similar to dynamic programming for bounding. Given an undirected graph $G = (V, E)$ where $V = \{v_1, v_2, \ldots, v_n\}$. The algorithm starts with the smallest subgraph containing only the last vertex in $V$ and then iteratively finds a maximum weight clique for subgraphs $V_n = \{v_n\}$, $V_{n-1} = \{v_{n-1}, v_n\}$, $V_{n-2} = \{v_{n-2}, v_{n-1}, v_n\}, \ldots$. This process ends up with the last subgraph $V_1$ which is the original graph to be solved and returns the maximum weight clique found. During the backtrack search of *Cliquer*, the information obtained in previously computed smaller graphs is used for better upper bounds for larger graphs. The performance of *Cliquer* greatly depends on the initial ordering of $V$. In *Cliquer*, vertices are sorted in descending order of weights, and vertices with the same weights are sorted by descending order of the sum of weights of adjacent vertices.

In [11], Kumlander proposed an exact algorithm based on a heuristic vertex coloring and a backtrack search for MWCP. The first step of this algorithm is to obtain a vertex coloring $c = \{C_1, C_2, \ldots, C_k\}$ of the graph $G = \{V, E\}$ and reorder the vertices first by color classes and then by weights inside each color class in ascending order. Then during the search process of the algorithm, this vertex coloring is frequently used to prune branches of the maximum weight clique search tree, since the vertex coloring upper bound computed as $\sum_{i=1}^{k} max\{w(u) | u \in C_i \cap S\}$ can be served as a more precise estimation on the bound of the subproblem $S$. A