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A color image reduction based on fuzzy transforms

Ferdinando Di Martino [a], Petr Hurtik [c], Irina Perfilieva [c], Salvatore Sessa [a,b,*]

[a] Università degli Studi di Napoli Federico II, Dipartimento di Architettura, Via Toledo 402, 80134 Napoli, Italy
[b] Università degli Studi di Napoli Federico II, Centro Interdipartimentale di Ricerca per l'Analisi e la Progettazione Urbana Luigi Pisciotti, Via Toledo 402, 80134 Napoli, Italy
[c] University of Ostrava, Institute for Research and Applications of Fuzzy Modeling, NSC IT4Innovations, 30. Dubna 22, 70103 Ostrava 1, Czech Republic

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ABSTRACT

We present a new method for color image reduction based on the concept of fuzzy transform. Any image in a single band can be considered as a fuzzy matrix which is subdivided into submatrices called blocks. Each block is compressed with various_compression rates by means of a fuzzy transform in two variables. We compare our method with recent three algorithms due to G. Beliakov, H. Bustince and D. Paternain based on the minimizing penalty functions defined over a discrete lattice. The quality of the reduced image is measured by the Mean Square Error (MSE) and Penalty function (PEN) obtained by comparing both magnified and original images. We also point out a threshold of the compression rate beyond which the MSE follows a linear trend and the corresponding loss of information is still acceptable.

1. Introduction

A fuzzy transform (shortly, *F*-transform) [16,17] is an operator which transforms a continuous function into a n-dimensional vector. Applications of the *F*-transforms were made in data analysis [7,8,14], image analysis [3–6,9,17–19] and comparisons with the fuzzy relation equations method and JPEG appear in [10–13]. In [1] three new color images reduction algorithms are presented and based on the optimizing penalty functions [2] defined over discrete product lattices. Furthermore the authors in [1] proved that these algorithms are better than other reduction algorithms based on appropriate resampling and *F*-transforms. Here we show that our algorithm based on decomposition of blocks reduced via *F*-transforms [3–5] gives better results than those obtained with the algorithms from [1]. In other words, as in [3–5], any image is divided into submatrices of equal dimensions, called blocks. Every block is reduced under a specific compression rate with a *F*-transform and reconstructed via a simple algorithm. The re-composition of these decompressed and magnified blocks gives an overall magnified image comparable with the original image. From the point of view of Granular Computing [15], we can also say that these blocks are the information granules which are then re-composed in accordance to some suitable criteria for giving the overall final information.

The quality of the reduced image is measured by the Mean Square Error (MSE) and the error based on Penalty function (PEN) obtained by comparing both magnified and original images. In addition, we develop a process to establish a compression rate threshold, through the analysis of the trend of the MSE with respect to the compression rates. Beyond this threshold the MSE follows a linear trend and the corresponding loss of information, due to reduction, is still acceptable. In Sections 2

* Corresponding author at: Università degli Studi di Napoli Federico II, Dipartimento di Architettura, Via Toledo 402, 80134 Napoli, Italy. Tel.: +39 0812538907; fax: +39 0812538909.
E-mail addresses: fdimarti@unina.it (F. Di Martino), petr.hurtik@osu.cz (P. Hurtik), irina.perfilieva@osu.cz (I. Perfilieva), sessa@unina.it (S. Sessa).

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and 3 we provide the definition of the F-transform in one and two variables, respectively. In Section 4 we present our reduction method. In Section 5 we present the results of our experimental study. Section 6 is conclusive.

2. *F*-transforms in one variable

Following the definitions and notations of [16], let $[a,b]$ be a closed interval, $n \geqslant 2$, and $x_1, x_2, \ldots, x_n$ be points of $[a,b]$, called nodes, such that $x_1 = a < x_2 < \cdots < x_n = b$. We say that an assigned family of fuzzy sets $A_1, \ldots, A_n$: $[a,b] \to [0,1]$ is a fuzzy partition of $[a,b]$ if the following conditions hold:

(1) $A_i(x_i) = 1$ for every $i = 1, 2, \ldots, n$;
(2) $A_i(x) = 0$ if $x \notin (x_{i-1}, x_{i+1})$, where we assume $x_0 = x_1 = a$ and $x_{n+1} = x_n = b$ by convenience of presentation;
(3) $A_i(x)$ is a continuous function on $[a,b]$;
(4) $A_i(x)$ strictly increases on $[x_{i-1}, x_i]$ for $i = 2, \ldots, n$ and strictly decreases on $[x_i, x_{i+1}]$ for $i = 1, \ldots, n-1$;
(5) $\forall x \in [a,b],\ \sum_1^n A_i(x) = 1$
The fuzzy sets $\{A_1(x), \ldots, A_n(x)\}$ are called basic functions. Moreover, we say that they form an uniform fuzzy partition if.
(6) $n \geqslant 3$ and $x_i = a + h \cdot (i-1)$, where $h = (b-a)/(n-1)$ and $i = 1, 2, \ldots, n$ (that is the nodes are equidistant);
(7) $A_i(x_i - x) = A_i(x_i + x)$ for every $x \in [0, h]$ and $i = 2, \ldots, n-1$;
(8) $A_{i+1}(x) = A_i(x-h)$ for every $x \in [x_i,\ x_{i+1}]$ and $i = 1, 2, \ldots, n-1$.

We limit ourselves only to the discrete case. We know that a given function f assumes assigned values in some points $p_1, \ldots, p_m$ of $[a,b]$. We assume that the set P of these points is sufficiently dense with respect to the fixed partition, i.e. for each $i = 1, \ldots, n$ there exists an index $j \in \{1, \ldots, m\}$ such that $A_i(p_j) > 0$. Then we can define the discrete F-transform of f with respect to $\{A_1, \ldots, A_n\}$ as the n-tuple $[F_1, \ldots, F_n]$ where each F_i is given by

$$F_i = \frac{\sum_{j=1}^m f(p_j) A_i(p_j)}{\sum_{j=1}^m A_i(p_j)} \tag{1}$$

for $i = 1, \ldots, n$. We call the discrete inverse F-transform of f with respect to $\{A_1, \ldots, A_n\}$ to be the following function defined in the same points $p_1, \ldots, p_m \in [a,b]$:

$$f_n^F(p_j) = \sum_{i=1}^n F_i A_i(p_j) \tag{2}$$

Analogously to Theorem 1, we have the following approximation theorem (cfr. [1, Theorem 5]):

Theorem 1. *Let f(x) be a function defined on a set $P = \{p_1, \ldots, p_m\} \subseteq [a,b]$. Then for every $\varepsilon > 0$, there exist an integer $n(\varepsilon)$ and a related fuzzy partition $\{A_1, \ldots, A_n(\varepsilon)\}$ of $[a,b]$ such that P is sufficiently dense with respect to $\{A_1, \ldots, A_{n(\varepsilon)}\}$ and for every $p_j \in P,\ j = 1, \ldots, m$,*

$$\left| f(p_j) - f_{n(\varepsilon)}^F(p_j) \right| < \varepsilon \tag{3}$$

holds true.

3. *F*-transforms in two variables

We can extend the above concepts to functions in two variables In the discrete case, let f the function assume determined values in some points $(p_j, q_j) \in [a,b] \times [c,d]$, where $i = 1, \ldots, N$ and $j = 1, \ldots, M$. Moreover, let the sets $P = \{p_1, \ldots, p_N\}$ and $Q = \{q_1, \ldots, q_M\}$ be sufficiently dense with respect to the chosen partitions, i.e. for each $i = 1, \ldots, N$ there exists an index $k \in \{1, \ldots, n\}$ such that $A_i(p_k) > 0$ and for each $j = 1, \ldots, M$ there exists an index $l \in \{1, \ldots, m\}$ such that $B_j(q_l) > 0$. In this case we define the matrix $[F_{kl}]$ to be the discrete F-transform of f with respect to $\{A_1, \ldots, A_n\}$ and $\{B_1, \ldots, B_m\}$ if we have for each $k = 1, \ldots, n$ and $l = 1, \ldots, m$:

$$F_{kl} = \frac{\sum_{j=1}^M \sum_{i=1}^N f(p_i, q_j) A_k(p_i) B_l(q_j)}{\sum_{j=1}^M \sum_{i=1}^N A_k(p_i) B_l(q_j)} \tag{4}$$

Then we can define the discrete inverse F-transform of f with respect to $\{A_1, \ldots, A_n\}$ and $\{B_1, \ldots, B_m\}$ to be the following function defined in the same points $(p_j, q_j) \in [a,b] \times [c,d]$, with $i \in \{1, \ldots, N\}$ and $j \in \{1, \ldots, M\}$, as

$$f_{nm}^F(p_i, q_j) = \sum_{k=1}^n \sum_{l=1}^m F_{kl} A_k(p_i) B_l(q_j) \tag{5}$$

It is not difficult to prove that the following generalization of Theorem 1 holds: