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Approximate solution of dual fuzzy matrix equations ☆

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ABSTRACT

In this paper, we propose a simple and practical method to solve the dual fuzzy matrix equation $A\tilde{x} + \widetilde{B} = C\tilde{x} + \widetilde{D}$, in which A, C are $m \times n$ matrices and $\widetilde{B}, \widetilde{D}$ are $m \times p$ LR fuzzy numbers matrices. By means of the arithmetic operations on LR fuzzy numbers space, the dual fuzzy matrix equation could be converted into two classical matrix equations, and the LR minimal fuzzy solution and the strong (weak) LR minimal fuzzy solutions of the dual fuzzy matrix equation are derived by solving two classical matrix equations based on the generalized inverses of matrices. Meanwhile, as a special case of the dual fuzzy matrix equation, the fuzzy solutions of the dual fuzzy linear systems are investigated. Finally, some numerical examples are given to illustrate the effectiveness of the proposed method.

1. Introduction

System of simultaneous linear matrix equations is an essential mathematical tool in science and technology. In practice, some of parameters of the system may be represented by fuzzy quantities rather than crisp ones, and hence it is necessary to develop mathematical theory and numerical schemes to handle fuzzy linear matrix systems. The $n \times n$ fuzzy linear system has been studied by several scholars. Buckley et al. [12] constructed solutions to the fuzzy equation $\widetilde{A}\tilde{x} = \tilde{b}$ when the elements of $\widetilde{A}$ and $\tilde{b}$ are triangular fuzzy numbers. They presented six solutions of the equation and shown that five of them are identical. Further works see [13,14]. In 1998, Friedman et al. [19] proposed a general method for solving fuzzy linear systems $Ax = b$ only whose right-hand side vector b is fuzzy by an embedding approach [31]. For the computation of fuzzy linear systems, Allahviranloo obtained the numerical solutions by using the iterative Jacobi method and Gauss-Siedel method, the Adomian method, and the successive over relaxation method, respectively in [4–6]. Abbasbandy et al. [1,2] solved a fuzzy system of equations using the LU decomposition method and the steepest decent method. Zheng et al. [29,33] discussed the $m \times n$ general fuzzy linear system and inconsistent fuzzy linear system by means of the generalized inverses of the coefficient matrix. For more research results see [15,16,20]. In 2010, Allahviranloo et al. [7] obtained symmetric fuzzy approximate solutions of fuzzy linear systems by solving a linear equation and a fuzzified interval equation which converted from the original fuzzy linear system $A\tilde{x} = \tilde{b}$. Subsequently, they investigated the maximal and minimal symmetric solutions of fully fuzzy linear systems $\widetilde{A}\tilde{x} = \tilde{b}$ by the same approach [8]. However, for a fuzzy matrix equation, which always has a wide use in control theory and control engineering, few works have been done over the past decades. In 2009, Allahviranloo et al. [9] discussed the fuzzy linear matrix equations (FLME) of the form $AXB = C$ where A and B are $m \times m$ and $n \times n$ real matrices, respectively, C is a given $m \times n$ fuzzy numbers matrix. By using the parametric form of the fuzzy number, they derived necessary and sufficient conditions for the existence of fuzzy solutions and designed a numerical procedure for calculating the

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solutions of the fuzzy matrix equations. Recently, Gong et al. [21–23] considered a class of fuzzy matrix equations $A\tilde{x} = \widetilde{B}$ by the block Gaussian elimination method and the undetermined coefficients method, and studied the inconsistent fuzzy matrix equation and its fuzzy least squares solutions. On the other hand, in many fuzzy linear systems [1,2,4–8,15,18,19,26,28,29,33] and fuzzy linear matrix equations [9,21–23] the known fuzzy vectors or matrices are denoted by triangular fuzzy numbers.

In 2000, Ma et al. [24] firstly discussed the dual fuzzy linear system by using the embedding approach. They remarked that the system $A_1x = A_2x + b$ is not equivalent to the system $(A_1 - A_2)x = b$, since there does not exist an element $\tilde{v}$ such that $\tilde{u} + \tilde{v} = \widetilde{0}$ for an arbitrary fuzzy number $\tilde{u}$. Later, Wang et al. [30] presented an iterative algorithm for solving dual linear system of the form $X = AX + U$, where A is a real $n \times n$ matrix, the unknown vector X and the constant U are vectors consisting of fuzzy numbers. Also, Muzziloi et al. [25] considered fuzzy linear systems of the form $A_1x + b_1 = A_2x + b_2$ with A_1, A_2 square matrices of fuzzy coefficients and b_1, b_2 fuzzy number vectors. In 2008, Abbasbandy et al. [3] proposed a numerical method for finding the minimal solution of the $m \times n$ general dual fuzzy linear system $AX + F = BX + C$ based on pseudo-inverse calculation.

The LR fuzzy number and its operations was first introduced by Dubois et al. [17]. We know that triangular fuzzy numbers are just specious cases of LR fuzzy numbers. In 2006, Dehgham et al. [16] discussed the computational methods for fully fuzzy linear systems whose coefficient matrix and the right-hand side vector are denoted by LR fuzzy numbers. In this paper we propose a simple and practical method to solve the dual fuzzy matrix equation $A\tilde{x} + \widetilde{B} = C\tilde{x} + \widetilde{D}$, in which A, C are $m \times n$ matrices and $\widetilde{B}, \widetilde{D}$ are $m \times p$ arbitrary LR fuzzy numbers matrices. By means of the arithmetic operations on LR fuzzy numbers space, the dual fuzzy matrix equation could be converted into two classical matrix equations, and the LR minimal fuzzy solution and the strong (weak) LR minimal fuzzy solutions of the dual fuzzy matrix equation are derived by solving two classical matrix equations based on the generalized inverses of matrices [10,11,27]. At the same time, we also discuss the dual fuzzy linear system using the proposed method in this paper, and show that the dual fuzzy matrix equation always has a LR minimal fuzzy solution. Finally, some numerical examples are given to illustrate the efficiency of proposed method in this paper. Since our model does not have parameter $r, 0 \leqslant r \leqslant 1$ and is made of two classical matrix equations, its solution becomes easy to implement. To the best of our knowledge, this is the first time in the literatures to investigate the general dual fuzzy matrix equation with LR fuzzy numbers. Meanwhile, as a special case of the dual fuzzy matrix equation, our method can be applied to solve a number of fuzzy linear systems and fuzzy linear matrix equations.

The structure of this paper is organized as follows. In Section 2 we review some basic concepts. In Section 3 the methods for solving the dual fuzzy matrix equation $A\tilde{x} + \widetilde{B} = C\tilde{x} + \widetilde{D}$ are discussed. In Section 4 we discuss the dual fuzzy linear systems using the similar methods. Numerical examples are given in Section 5.

2. Preliminaries

In this section, some basic definitions (e.g. LR fuzzy numbers, LR fuzzy matrix, generalized inverses of matrix, and dual fuzzy matrix equations) and arithmetic operations of LR fuzzy numbers are presented.

Definition 2.1 [32]. A fuzzy number is a fuzzy set like $u : R \to I = [0, 1]$ which satisfies:

(1) u is upper semicontinuous,
(2) u is fuzzy convex, i.e., $u(\lambda x + (1-\lambda)y) \geqslant \min\{u(x), u(y)\}$ for all $x, y \in R, \lambda \in [0, 1]$,
(3) u is normal, i.e., there exists $x_0 \in R$ such that $u(x_0) = 1$,
(4) $\mathrm{supp}\,u = \{x \in R | u(x) > 0\}$ is the support of the u, and its closure cl $(\mathrm{supp}\,u)$ is compact.

Let E^1 be the set of all fuzzy numbers on R.

Definition 2.2 [17]. *A* fuzzy number $\widetilde{M}$ is called a LR fuzzy number if

$$\mu_{\widetilde{M}}(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right), & x \leqslant m, \quad \alpha > 0, \\ R\left(\frac{x-m}{\beta}\right), & x \geqslant m, \quad \beta > 0, \end{cases}$$

where m is the mean value of $\widetilde{M}$, and α and β are left and right spreads, respectively. The function $L(\cdot)$, which is called left shape function satisfying:

(1) $L(x) = L(-x)$,
(2) $L(0) = 1$ and $L(1) = 0$,
(3) $L(x)$ is nonincreasing on $[0, \infty)$.

The definition of a right shape function $R(\cdot)$ is usually similar to that of $L(\cdot)$.