Information Sciences 266 (2014) 171–185

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Non-fragile H_∞ filter design for discrete-time fuzzy systems with multiplicative gain variations

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A R T I C L E   I N F O

Article history:
Received 4 April 2011
Received in revised form 22 June 2013
Accepted 23 August 2013
Available online 2 September 2013

Keywords:
T–S fuzzy systems
Non-fragile H_∞ filters
Multiplicative gain variations
Filter state transformation
Linear matrix inequalities (LMIs)

A B S T R A C T

This paper investigates the problem of non-fragile H_∞ filter design for discrete-time Takagi–Sugeno (T–S) fuzzy systems. The filter to be designed is assumed to have two type of multiplicative gain variations. By introducing additional matrix variables, a relaxed H_∞ filtering analysis criterion is presented for the fuzzy system based on the fuzzy Lyapunov function. A filter state transformation approach is adopted to design the non-fragile H_∞ filter, and the filter gains are given in terms of the feasible solutions of a set of linear matrix inequalities (LMIs). A simulation example is given to show the efficiency of the proposed methods.

1. Introduction

It is well known that the problem of H_∞ filtering is both theoretically and practically important in control and signal processing [10,29]. The main advantage of H_∞ filtering lies in that no statistical assumption on the noise signals is needed, and thus, it is more general than classical Kalman filtering [9]. Moreover, the H_∞ filter is designed by minimizing signal estimation error for the bounded disturbances and noises of the worst cases, which is more robust than the Kalman filter [20].

During the past several years, fuzzy systems of the Takagi–Sugeno (T–S) model [27] have attracted great interests from the control community. Much effort has been made in the control literature to investigate T–S fuzzy systems and various techniques have been developed for the stability analysis and controller synthesis of T–S fuzzy systems [5,7,12,15,18,19,23–25]. For the fuzzy H_∞ filtering problem based on T–S fuzzy models, some important results have been obtained. To mention a few, for singularly perturbed fuzzy systems with pole placement constraints [1], for communication fuzzy systems using fuzzy interpolation [3], for fuzzy systems with noises in finite frequency domain [4], for fuzzy systems based on the notion of quadratic stability [6,32], for fuzzy systems with intermittent measurements [11], for discrete-time fuzzy systems using fuzzy Lyapunov functions [34,35], for networked fuzzy systems (NCSs) [13], for time-delay fuzzy systems [17], for fuzzy systems with D stability constraints [21], for singularly perturbed fuzzy systems with the consideration of improving the bound of singular-perturbation parameter [30], for interconnected fuzzy systems [33].

On the other hand, the non-fragile control and filtering problems have been attractive topics in theory analysis and practical implement. The non-fragile concept is proposed to this new problem: how to design a controller or filter that will be insensitive to some error in gains [14,31]. The problem of non-fragile H_∞ filtering for continuous-time T–S fuzzy systems

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0020-0255/$ - see front matter
http://dx.doi.org/10.1016/j.ins.2013.08.052

with respect to additive norm-bounded filter gain variations has been investigated in [2]. By using the quadratic Lyapunov function approach and introducing slack matrix variables, [2] proposed LMI-based conditions for designing the non-fragile H_∞ filter. Nevertheless, the result in [2] does not address the design of non-fragile H_∞ filter for discrete-time T–S fuzzy systems with multiplicative gain variations.

This paper is concerned with the problem of non-fragile H_∞ filter design for discrete-time T–S fuzzy systems. In contrast to the existing results, this paper contributes in three aspects: (i) A non-fragile fuzzy H_∞ filter with two type of multiplicative gain variations will be designed. (ii) Additional matrix variables are introduced to decouple the Lyapunov matrix and the system matrices, which make the filter design feasible and will bring less conservative results. (iii) A filter state transformation approach is adopted to design the non-fragile fuzzy filter and LMI-based design conditions, which guarantee the H_∞ performance of the filter error system, are derived.

The remainder of this paper is organized as follows. The problem formulation and preliminaries are presented in Section 2. In Section 3, the H_∞ filtering analysis for the filtering error system is given, then the result is employed to design the non-fragile H_∞ filter. An illustrative example is presented in Section 4, and we conclude the paper in Section 5. The proof of Lemma 3, Theorems 4 and 6 is given in Appendices A, B and C, respectively.

Notations: For a matrix A, A^T and A^{-1} denote its transpose and inverse if it exists, respectively. The symbol $(*)$ induces a symmetric structure in LMIs. $l_2[0,\infty)$ is the space of square-integrable vector functions over $[0,\infty)$.

2. Problem formulation and preliminaries

Consider the following discrete-time nonlinear system represented by T–S fuzzy dynamic model, in which the ith rule is described as follows [27]:

$$\begin{aligned}
&\textit{Plant Rule } i: \ \textit{if } \ \xi_1(k) \ \textit{is} \ M_{1i} \ \textit{and} \ldots \xi_p(k) \ \textit{is} \ M_{pi}\\
&\quad \textit{then } x(k+1) = A_i x(k) + B_i w(k)\\
&\quad\quad y(k) = C_i x(k) + D_i w(k)\\
&\quad\quad z(k) = L_i x(k)
\end{aligned} \tag{1}$$

where $x(k) \in R^n$ is the state variable, $w(k) \in R^v$ is the noise signal that is assumed to be the arbitrary signal in $l_2[0,\infty)$, $z(k) \in R^q$ is the signal to be estimated, $y(k) \in R^f$ is the measurement output, $\xi_1(k), \xi_2(k), \ldots, \xi_p(k)$ are premise variables vector and measurable, M_{di}, $i = 1,2,\ldots,r$, $d = 1,2,\ldots,p$ are the fuzzy sets, r is the number of fuzzy rules. $A_i \in R^{n\times n}$, $B_i \in R^{n\times v}$, $C_i \in R^{f\times n}$, $D_i \in R^{f\times v}$, $L_i \in R^{q\times n}$ for $i = 1,2,\ldots,r$ are system matrices.

Denote

$$\omega_i(\xi(k)) = \prod_{d=1}^{p} M_{di}(\xi_d(k)), \quad i = 1,2,\ldots,r \tag{2}$$

where $\xi(k) = (\xi_1(k), \xi_2(k), \ldots, \xi_p(k))$, $M_{di}(\xi_d(k))$ is the grade of membership function of $\xi_d(k)$ in M_{di}.

It is assumed in this paper that

$$\omega_i(\xi(k)) > 0, \ \sum_{i=1}^{r}\omega_i(\xi(k)) > 0, \quad i = 1,2,\ldots,r \tag{3}$$

Let

$$h_i(\xi(k)) = \frac{\omega_i(\xi(k))}{\sum_{j=1}^{r}\omega_j(\xi(k))}, \quad i = 1,2,\ldots,r \tag{4}$$

Then

$$h_i(\xi(k)) \geqslant 0, \ \sum_{i=1}^{r} h_i(\xi(k)) = 1, \quad i = 1,2,\ldots,r \tag{5}$$

The T–S fuzzy model (1) is inferred as follows:

$$\begin{aligned}
x(k+1) &= A(h)x(k) + B(h)w(k)\\
y(k) &= C(h)x(k) + D(h)w(k)\\
z(k) &= L(h)x(k)
\end{aligned} \tag{6}$$

where

$$\begin{aligned}
&A(h) = \sum_{i=1}^{r} h_i(\xi(k))A_i, \ B(h) = \sum_{i=1}^{r} h_i(\xi(k))B_i\\
&C(h) = \sum_{i=1}^{r} h_i(\xi(k))C_i, \ D(h) = \sum_{i=1}^{r} h_i(\xi(k))D_i, \ L(h) = \sum_{i=1}^{r} h_i(\xi(k))L_i
\end{aligned}$$