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The Cournot production game with multiple firms under an ambiguous decision environment

Jr-Fong Dang [a], I-Hsuan Hong [b,*,1], Jing-Ming Lin [a]

[a] Department of Industrial Engineering & Management, National Chiao Tung University, 1001 Ta Hsueh Road, Hsinchu 300, Taiwan
[b] Institute of Industrial Engineering, National Taiwan University, 1 Section 4 Roosevelt Road, Taipei 106, Taiwan

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ABSTRACT

The recent economic recession has added more uncertainty to industries' decision processes regarding production quantity. Moreover, due to the nature of ambiguous information, manufacturers often fail to achieve precise assessments of the parameters of market demand, production cost functions, etc. This paper develops the Cournot production game with multiple firms in an ambiguous decision environment, where the form of ambiguity is described by a set of fuzzy parameters. Our model applies the weighted center of gravity method (WCoG) to defuzzify the fuzzy profit function considering firms' control parameters. The resulting outcomes are in the form of matrix representations. We also analyze the effect of firms' control parameters on outcomes. The results indicate that a firm's fuzzy profit function plays an important role in economic interpretation. To investigate the effect of parameter perturbations on firms' outcomes, we also conduct the sensitivity analysis.

1. Introduction

A competitive global economy presents opportunities for new approaches to solve for the production quantity of decision makers who are forced to operate in ambiguous environments. Intuitively, the decisions of multiple agents will affect the payoffs of others. Thus, the conventional Cournot game is one of several methods commonly applied to analyze precise scenarios when exact model parameters are available. Liang et al. [20] categorize these as "precision-based models" because all data are required to be precise. Yet, real-world decision makers typically are hampered due to a lack of data and/or the imprecision of the available information concerning the behavior of other decision makers (see [17]), and both conditions make it difficult to apply the Cournot game to real problems. Therefore, the excess of capacity observed in the recent economic recession motivated us to develop a Cournot game with ambiguous parameters.

Employing the available game-theoretical models for decision making can be difficult due to the uncertainty of data in the form of randomness and ambiguity. Even though the literature has proposed many stochastic game-theoretical models [3,11,15,27,30], these models only consider the probabilistic type of uncertainty. In practice, however, the probability distribution may not be available or the limited number of data points cannot provide exact estimates of a manufacturer's variable cost since the procurement costs can fluctuate over time. Thus, the fuzzy sets theory pioneered by Zadeh [36] becomes useful because it can mathematically model the vagueness and impression of human cognitive processes, e.g., the phrase "around x dollars", to describe a cost that can be regarded as a fuzzy number $\tilde{x}$.

* Corresponding author. Tel.: +886 2 3366 9507; fax: +886 2 2372 5856.
E-mail address: ihong@ntu.edu.tw (I-H. Hong).

[1] The corresponding author holds a joint appointment with the Department of Mechanical Engineering, National Taiwan University.

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Unlike the literature which considers only zero-sum games, Dang and Hong [10] indicate that there are two streams of fuzzy games: fuzzy matrix games and fuzzy non-cooperative games. The matrix game can be solved by the linear programming method based on the duality (see [4,28,29]). Maeda [22,23] studies two-person zero-sum games and bimatrix games with fuzzy payoffs and applies α-cut, possibility, and necessity theories to introduce two concepts of the equilibrium derived by the mathematical programming. Liu and Kao [21] obtain the upper and lower bound values of a matrix game by utilizing a pair of two-level mathematical programming.

Yao and Wu [31], who probably initiated the fuzzy non-cooperative game involving fuzzy data, apply the ranking method transforming fuzzy numbers for comparison to defuzzify the demand and supply functions so that consumer surplus and producer surplus can be calculated in a conventional manner. Their method of transforming fuzzy numbers to crisp values is also utilized to construct the monopoly model [6]. Yao and Wu [32] discuss the best price of two mutual complementary merchandises in a fuzzy sense. Yao and Chang [33] obtain the optimal quantity for maximizing the profit function whose parameters are fuzzy numbers. Yao and Shih [34] derive the membership function of the profit function when the optimal quantity occurs. Liang et al. [20] propose a duopoly model considering only fuzzy costs to obtain the optimal quantity of each firm. Dang and Hong [10], who highlight an unreasonable occurrence of a negative equilibrium quantity, and the limited flexibility for modification of the ranking method in fuzzy modeling in previous studies, propose a fuzzy Cournot game with rigorous definitions ensuring a positive equilibrium quantity and with a controlling mechanism. However, for model simplicity, Dang and Hong [10] restrict themselves to constant parameters of the controlling mechanism and to a duopoly production game. Furthermore, Guo [12] initially proposes a one-shot decision approach to solve for the Cournot equilibrium, where the solution procedure can be separated in two steps. In the first step, a decision maker can be categorized as passive, normal or active attitude depicted by the satisfaction level that relates to his/her own profits. At the second step, a solution procedure is then performed to seek for the Cournot equilibrium quantity, where the difference between the possibility of the demand uncertainty and the satisfaction level is within a pre-specified tolerance. Guo et al. [13] further extend the method proposed in Guo [12] to a duopoly market with asymmetric possibilistic information describing the demand uncertainty only known by one firm. This paper differs from (Guo [12] and Guo et al. [13]) in deriving the Cournot equilibrium quantity with less computational manipulations which simplify the analysis of the model. Furthermore, the proposed model in this paper comprehensively considers the uncertainty resulted from demand and cost functions.

This paper makes two contributions to the literature. First, we introduce a method solving for the equilibrium quantity of each competing firm in a competitive market with multiple firms, where the demand and cost functions are characterized by the form of ambiguity described by a set of fuzzy parameters, and the weighted center of gravity (WCoG) proposed by Bender and Simonovic [5] is used to defuzzify a firm's profit function into a crisp value. For simplicity, we assume that a firm's demand function and cost function take the form of linearity with fuzzy parameters, in order to obtain managerial insights with less analytical complexity (see [10,33]). Second, we investigate the impact of the perturbation of uncertainty on the resulting outcomes. For instance, we note that the fuzzy profit function and firms' control parameters play key roles in analyzing the perturbation of equilibrium quantity.

The remainder of this paper is organized as follows. In Section 2, we introduce the concepts and definitions of the proposed model. Section 3 addresses the Cournot production game and presents the proposed method to solve for the equilibrium quantity of each firm in an ambiguous environment where multiple firms exist in a competitive market. In Section 4, we analyze the resulting outcomes and discuss several valuable managerial insights.

2. Preliminary

This section presents the fuzzy sets theory and weighted center of gravity (WCoG) which are integral to this paper.

2.1. Fuzzy sets theory

The fuzzy sets theory initiated by Zadeh [36] attempts to analyze and to solve problems with a source of ambiguity called fuzziness. In the following, we introduce the definitions and notations of triangular fuzzy numbers, the extension principle, and the WCoG method.

2.1.1. Triangular fuzzy numbers

A popular type of fuzzy numbers is the triangular type because it is easy to handle arithmetically and has intuitive interpretation [9,29,30]. Dağdeviren and Yüksel [9] indicate that using triangular fuzzy numbers has proven efficient for calculating a decision making problem. Petrovic et al. [25] and Giannoccaro et al. [14] show that triangular fuzzy numbers are the most suitable for modeling market demand in a fuzzy sense (see [1,2,16,24] for other applications of triangular fuzzy numbers). The membership function $\mu_{\widetilde{A}}(x)$ of a triangular fuzzy number $\widetilde{A}$ can be defined by

$$\mu_{\widetilde{A}}(x) = \begin{cases} \frac{x-m_A+l_A}{l_A}, & m_A - l_A \leqslant x \leqslant m_A \\ \frac{m_A+r_A-x}{r_A}, & m_A \leqslant x \leqslant m_A + r_A \\ 0, & \text{otherwise}, \end{cases} \tag{1}$$