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Robust control on saturated Markov jump systems with missing information

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ABSTRACT

In this paper, a robust H_∞ controller is designed for saturated Markov jump systems with uncertainties and time varying transition probabilities. The time-varying transition probability uncertainty is described as a polytope set. Stochastic stability is analyzed for the underlying systems by Lyapunov function approach and a sufficient condition is derived to design controllers such that the resulting closed-loop system is stochastically stable and a prescribed H_∞ performance is also achieved. Furthermore, the attraction domain of this Markov jump system is estimated and evaluated. A simulation example is given to show the effectiveness of the developed techniques.

1. Introduction

Markov jump systems (MJSs) were introduced by Krasovskii and Lidskii in 1961 [8]. This kind of systems have received much attention among scholars and engineers due to their comprehensive application in many practical systems, such as in manufacturing systems, economic systems, electrical systems and communication systems [5,15]. In recent years, much work has been done on MJSs as they can be used to describe systems subject to abrupt variation in their structures or parameters, which caused by failures of subsystems, sudden environmental changes, and system noises, and a large variety of control problems have been investigated, such as stochastic stability and stabilization [1,9,23], fault detection and filtering [4,12,16,20–22], etc. Considering Markov jump systems with uncertain transition probabilities, some work has been conducted, see for example, [19] and the references therein.

On another research front line, a great amount of effort has been devoted to systems with saturation nonlinearity. As well known, in practice, almost all actuators have their limited working region, if the input of system exceeds the maximal capacity or lower than the minimal capacity, then, it will lead to the actuator saturation nonlinearity, and such nonlinearity is a complicated nonlinear constraint in system, and occurs very common in biochemistry system, networked control systems and communication systems. It is also a well recognized fact that this nonlinearity degrades system performance and even leads a stable system to an unstable one, and a linear system to a nonlinear one. Therefore, actuator saturation is probably the most dangerous nonlinearity in many dynamical systems. It is a challenge work to solve control problem in the presence of actuator saturation nonlinearity constraints. Some attempts have been made on control problems of linear stochastic

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systems and nonlinear stochastic systems with actuator saturation nonlinearity, see for example, [11,13,14] and the references therein.

It is worth mentioning that most the works mentioned above are all under the assumption that the systems must satisfy time-invariant Markov process or Markov chain, in which transition probabilities are constants. However, this assumption is not realistic in many situations, where the transition probability of Markov jump system is a time-dependent and time-varying matrix. One typical example is networked systems [6,10,18,24,25], in which packet dropouts and network delays evolve in Markov chains or Markov processes. However, internet delay or packet dropouts are different in different time periods, which will bring in time-varying transition probabilities as the transition rates vary through the whole working region. One typical example is the VOTL (vertical take-off landing) helicopter system, all the transition probabilities of the process among multiple airspeeds are not fixed when the surrounding environment is changed. One feasible and reasonable assumption is to use a polytope set to describe this characteristics of uncertainties caused by time-varying transition probabilities. The main reason is that although the transition probability of the Markov process is not exactly known, but one can evaluate some values in some points, and assume that the time-varying transition probabilities evolve in this polytope, which belongs to a convex set. The polytope set has been successfully used in some control problems [3], however, there is still limited systematic work on such Markov system with time-varying transition probabilities, not to mention control problems on saturated Markov systems, which is important and stimulates us on the current work.

In this paper, we will design H_∞ controllers for a class of saturated Markov systems with parameter uncertainty and partially known transition probabilities. Moreover, for actuator saturation nonlinearity, we will also study the estimation of attraction domain of such system, while the system states starting from such attraction domain will remain in it, which the problem will be transformed to an optimization problem in order to make the largest domain of attraction. The rest of the paper is organized as follows: Problem statement and preliminaries are given in Section 2. Stochastic stability analysis for the system is presented in Section 3. An H_∞ controller is designed in Section 4 for the systems. Estimation of attraction domain is made in Section 5. A numerical example is provided in Section 6 to show the effectiveness of our approach. Finally, some concluding remarks are drawn in Section 7.

Notation. Throughout the paper, the notation R^n stands for a n-dimensional Euclidean space, the transpose of a matrix is denoted by A^{T}, $E\{\cdot\}$ denotes the mathematical expectation of the stochastic process or vector, $L_2^n[0,\infty]$ stands for the space of n-dimensional square integrable function vector over $[0,\infty]$, a positive-definite matrix is denoted by $P > 0$, I is the unit matrix with appropriate dimension, and $*$ means the symmetric term in a symmetric matrix, $\sigma(\cdot)$ is the standard saturation function with appropriate dimensions.

2. Problem statement and preliminaries

Consider a probability space (M,F,P) where M, F and P represent the sample space, the algebra of events and the probability measure defined on F, respectively, then the following discrete-time Markov jump systems (MJSs) are considered in this paper:

$$\begin{cases} x_{k+1} = A(r_k)x_k + B(r_k)\sigma(u_k) + C(r_k)w_k + g(x_k, r_k) \\ z_k = D(r_k)x_k + E(r_k)w_k \end{cases} \tag{2.1}$$

where $A(r_k)$, $B(r_k)$, $C(r_k)$, $D(r_k)$ and $E(r_k)$ are mode-dependent constant matrices with appropriate dimensions at the working instant k, $x_k \in R^n$ is the state vector of the system, $u_k \in R^m$ is control input, $\sigma(u_k)$ is defined as the standard saturation function in this paper, where $\sigma(u_k) = [\sigma(u_{1k}) \quad \sigma(u_{2k}) \quad \ldots \quad \sigma(u_{mk})]^{\mathrm{T}}$ and $\sigma(u_{lk}) = \{sign(u_{lk})min\{1, \mid u_{lk} \mid\}, l = 1, \ldots, m\}$. $z_k \in R^p$ is the controlled output vector of the system, $w_k \in L_2^q[0,\infty]$ is the external disturbance vector of the system, $g(\cdot)$ is time-dependent and norm-bounded uncertainties, $\{r_k, k \geqslant 0\}$ is the concerned time-discrete Markov stochastic process which takes values in a finite state set $\Lambda = \{1,2,3,\ldots,N\}$, and r_0 represents the initial mode.

The transition probability matrix is defined as:

$$\Pi(k) = \{\pi_{ij}(k)\}, \quad i,j \in \Lambda$$

$\pi_{ij}(k) = P(r_{k+1} = j \mid r_k = i)$ is the transition probability from mode i at time k to mode j at time $k+1$, which satisfies $\pi_{ij}(k) \geqslant 0$ and $\sum_{j=1}^{N}\pi_{ij}(k) = 1$.

It is noted that the robust H_∞ control problem is considered for a class of Markov jump systems (2.1) with both parameter uncertainty and time-varying transition probabilities, such transition uncertainty is assumed to evolves in a polytope.

For given vertices Π^s, $s = 1,\ldots,w$, the time varying transition matrix $\Pi(k)$ can be described as

$$\Pi(k) = \sum_{s=1}^{w}\alpha_s(k)\Pi^s$$

where

$$0 \leqslant \alpha_s(k) \leqslant 1, \quad \sum_{s=1}^{w}\alpha_s(k) = 1$$