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Information acquisition processes and their continuity: Transforming uncertainty into risk

Debora Di Caprio [a,b], Francisco J. Santos-Arteaga [c], Madjid Tavana [d,e,*]

[a] Department of Mathematics and Statistics, York University, Toronto M3J 1P3, Canada
[b] Polo Tecnologico IISS G. Galilei, Via Cadorna 14, 39100 Bolzano, Italy
[c] Departamento de Economía Aplicada II and Instituto Complutense de Estudios Internacionales, Universidad Complutense de Madrid, Campus de Somosaguas, 28223 Pozuelo, Spain
[d] Business Systems and Analytics Department, Lindback Distinguished Chair of Information Systems and Decision Sciences, La Salle University, Philadelphia, PA 19141, USA
[e] Business Information Systems Department, Faculty of Business Administration and Economics, University of Paderborn, D-33098 Paderborn, Germany

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ABSTRACT

We propose a formal approach to the problem of transforming uncertainty into risk via information revelation processes. Abstractions and formalizations regarding information acquisition processes are common in different areas of information sciences. We investigate the relationships between the way information is acquired and the continuity properties of revelation processes. A class of revelation processes whose continuity is characterized by how information is transmitted is introduced. This allows us to provide normative results regarding the continuity of the information acquisition processes of decision makers (DMs) and their ability to formulate probabilistic predictions within a given confidence range.

1. Introduction

In the context of decision-making under risk, each outcome occurs with a known probability, that is, the probability function defined on a given set of outcomes *S* is assumed to be known to the decision maker (DM). In the realm of decision-making under uncertainty, the DM faces a set of possible outcomes, each of them happening with an unknown probability (see, e.g., [21]).

In general, decision theory studies uncertainty and risk separately, e.g., [13]. We study the problem of how an uncertain situation may be transformed into a risky one. Moving from an uncertain situation to a risky one implies that some information must be revealed in the process. However, information theory concentrates mainly on the stochastic properties of information without considering the information revelation/acquisition process as a topological object, e.g., [5].

We propose a novel purely set-theoretical and topological approach to information revelation through time that allows a DM to move from an initial situation of uncertainty to a final situation of risk. In our framework, *revealing (or transmitting) information through time* means providing the DM with the information necessary to determine the true probability (probability

* Corresponding author at: Business Systems and Analytics Department, Lindback Distinguished Chair of Information Systems and Decision Sciences, La Salle University, Philadelphia, PA 19141, USA. Tel.: +1 215 951 1129; fax: +1 267 295 2854.
E-mail addresses: dicaper@mathstat.yorku.ca (D. Di Caprio), fransant@ucm.es (F.J. Santos-Arteaga), tavana@lasalle.edu (M. Tavana).
URL: http://tavana.us/ (M. Tavana).

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mass or probability density according to the nature of the set of outcomes) of each outcome. Throughout the paper, the theoretical analysis will concentrate on the *revelation* perspective of the *information* process. However, when presenting the managerial interpretation of the results we will consider the *information acquisition* perspective of the DM. Both approaches are equivalent within the current environment. That is, the object of study is the process of information transmission, which can be analyzed either from the perspective of the transmitter or that of the receiver (the DM). In our setting, the properties of the process studied do not depend on the perspective considered.

Letting $T = [0, 1]$ represent the time interval during which information is revealed, we will consider revelation processes with the following characteristics. At time 0, no information is available to the DM, who faces the highest uncertainty: any probability function definable on S could be the true one. As time goes by, the DM receives additional information until a certain time $t \leqslant 1$ when all the information is revealed. Clearly, the more information that is revealed the less uncertainty is faced by the DM. As the amount of information on the true probability function increases, the DM excludes more and more elements from the set of all possible probability functions on S. At time $t \leqslant 1$, when all the information becomes available, the DM is left only with the true probability function. Thus, information being fully revealed is equivalent to uncertainty being completely transformed into risk.

To formalize the proposed problem we define information revelation processes by means of multifunctions from the time interval T to the set PR of all the probability functions defined on S. A revelation process is continuous if it satisfies both lower and upper semi-continuity as a multifunction. No restriction is imposed initially on when and how information is transmitted and the full information revealed. The main objective of the paper is to investigate the relationship between when and/or how information is transmitted and the continuity properties of revelation processes.

We show that the continuity – both lower and upper semi-continuity – of revelation processes is determined by how information is transmitted through time and not by when it is transmitted. In this regard, we introduce a class of revelation processes whose continuity is characterized by how information is transmitted. These revelation processes present a desirable and natural contraction-like pattern towards the true probability function. Finally, we characterize a subclass of these revelation processes satisfying a useful reflecting property.

The main results are intuitively introduced through several examples explicitly discussing different information transmission situations. These examples demonstrate the applications of our framework to management and information sciences.

1.1. Theoretical motivation

The last decades have witnessed a significant amount of effort on theoretical and practical explorations of intelligent techniques and soft computing that deal with the uncertainty inherent in scientific, engineering and business decision-making processes. Information acquisition has become an important multi-disciplinary domain of information sciences receiving increasing attention (see [6,25] for special issues on this research area and reviews of the corresponding literature).

When dealing with an abstract intangible object such as the flow of information and its acquisition, a formal approach leads naturally to the fields of topology and set theory, see [31,14]. In particular, the formal modelization of uncertain and partial information environments within the information sciences builds on the developments of fuzzy set theory [39,38], possibility theory [9,40], probability and belief structures [30,35,36], and rough set theory [28,37,38]. Generally, the convergence properties of uncertain series are studied from a measure-theoretical perspective, e.g., [20,34]. In the same way, the complex interactions defining the dynamics of information acquisition processes have been analyzed to measure theoretical terms [19]. In this paper, we take the convergence of the process as given and concentrate on its continuity based on the information acquisition behavior of the DMs.

We follow a topological/set-theoretical approach to study the process of information acquisition within an initially uncertain environment. Our approach differs significantly from the standard rough set theoretical and information theoretical ones, summarized by [7,20], and has been chosen for two main reasons. First, it allows us to characterize information as a tractable object whose properties may be defined and analyzed. Second, we are able to derive properties that relate directly to the information acquisition behavior of DMs. In particular, the characterization of the information acquisition process as a topological object allows us to study the effects of its continuity in making reasonably sound decisions. The main result of the current paper, besides the formal characterization of information processes, is how the way DMs acquire information and learn about an initially uncertain variable has important consequences regarding their ability to make accurate probabilistic judgments. That is, if the information acquisition process does not satisfy a key continuity condition [refer to the reflecting Property ($\triangleright\triangleleft$) in Section 6], the ability of DMs to formulate probabilistic predictions within a given confidence range can only be guaranteed in the limit time defining the process. As a result, DMs may continue acquiring information after being able to generate sufficiently accurate judgments, which is suboptimal both in behavioral and information acquisition terms.

Finally, our topological interpretation of an information revelation process as a multifunction H from the time interval T to the set PR of all the probability functions defined on S allows for both: its interpretation in rough set-theoretical terms and potential fuzzy set-theoretical expansions. Refer, in particular, to [38] for a description of the main differences between both theories. More precisely, rough sets could be used to describe the indiscernibility among the probability functions in the image sets of our revelation processes, while the use of fuzziness would constitute a generalization of the dynamical evolution of these image sets. We will further elaborate on these potential extensions in the conclusion section.