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# Method of multi-criteria group decision-making based on cloud aggregation operators with linguistic information

Wang Jian-qiang *, Lu Peng, Zhang Hong-yu, Chen Xiao-hong

*School of Business, Central South University, Changsha 410083, China*

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ABSTRACT

The cloud model, which can synthetically describe the randomness and fuzziness of qualitative concepts and implement uncertain transformations between a qualitative concept and its quantitative instantiations, has attracted considerable attention from researchers studying multi-criteria group decision-making problems involving linguistic information. In this paper, some operations of clouds and several new aggregation operators are proposed. These include the cloud weighted arithmetic averaging (CWAA) operator, cloud-ordered weighted arithmetic averaging (COWA) operator, and cloud hybrid arithmetic (CHA) operator. The conversion between linguistic variables and clouds is introduced. Based on this conversion, a linguistic multi-criteria group decision-making method is developed. In this method, linguistic variables are first converted into clouds and then aggregated using cloud aggregation operators. The proposed method is then compared to the existing methods to confirm its feasibility and rationality.



## 1. Introduction

Due to the complexities of objects and the vagueness of the human mind, it is more appropriate for decision-makers to use linguistic descriptors than other descriptors to express their assessments in the actual process of multi-criteria decision-making (MCDM) [2]. However, natural languages usually involve ambiguity and uncertainty, so it is difficult to form an exact definition of linguistic information. Among the uncertainties involved in natural language, randomness and fuzziness are the two most important aspects [20]. The fuzziness of a concept mainly refers to uncertainty regarding the range of extension of that concept, and the randomness of a concept means that any concept is related to the external world in various ways, and is not an isolated fact [20]. Most often, the fuzziness and randomness of concepts are tightly related and inseparable, and both terms can be used to describe the uncertainty of natural languages. Although the uncertainty of natural languages can make it difficult to make exact evaluations of objects, the application of the qualitative concept makes communication among human beings comprehensive and cognitive [9].

Since the introduction of fuzzy sets, the fuzziness of the qualitative concept has been solved to some extent. Among the various membership functions of fuzzy sets, the normal (Gauss) membership function is considered one of the most suitable for qualitative concepts [20]. Regarding the approximation of large numbers of random phenomena, probability theory is the main mathematical tool used to address randomness, in which normal distribution is the most important probability distribution [20]. During the process of linguistic decision-making, a linguistic term given by different decision-makers

* Corresponding author. Tel.: +86 073188830594.
*E-mail address:* jqwang@csu.edu.cn (J.-Q. Wang).

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may take on different meanings. For example, decision-maker A thinks 80% fulfillment of a task is ''good'' but decision-maker B thinks that less than 85% fulfillment of the same task cannot be considered ''good'' with the same linguistic term scale. In this way, when considering the degree of certainty of an element belonging to a qualitative concept in a specific universe, it is more feasible to allow a stochastic disturbance of the membership degree encircling a determined central value than to allow a fixed number [20]. The cloud model, which describes a qualitative concept, is based on the foundation of probability theory and fuzzy set theory, detailing the fuzziness of a qualitative concept with normal (Gauss) membership function, and the randomness with normal distribution. The cloud model also depicts the fuzziness and randomness of a qualitative concept with three numerical characteristics perfectly, in such a way that objective and interchangeable transformation between qualitative concepts and quantitative values becomes possible [22,26]. In this way, the cloud model offers a new method of linguistic decision-making.

Linguistic decision-making, with its closeness to reality, has been widely applied in various fields, such as mechanical design [19], evaluation of nuclear safeguards [39], web community [1], and emergency management [18]. Furthermore, unbalanced linguistic terms [4,35], multi-granularity linguistic terms [38,64] and several novel solutions to uncertain linguistic decision-making problems [30,51,56] have been studied recently. Computing with words (CWs) is the key to transforming linguistic variables into quantitative values, so it has attracted considerable attention [15,31,32]. With the great efforts of researchers in the past few decades, linguistic decision-making methods can be classified into three main types according to the ways of CW, i.e., linguistic computational model based on membership functions [6,11], linguistic symbolic model based on ordinal scales [10,13] and 2-tuple linguistic model [31,45]. The linguistic symbolic model and the 2-tuple linguistic model cannot produce a clearer description of either fuzziness or randomness. The linguistic computational model can describe fuzziness but not randomness. However, the cloud model not only describes the fuzziness and randomness of linguistic terms but also renders the transformation between the quantitative values and qualitative concepts more objective and interchangeable. In this way, the cloud model is of great value in linguistic MCDM problems.

It has been more than ten years since Li introduced the cloud model [25,27]. Since then, a theoretical system of this model has been gradually forming. A good number of specialized studies have been performed, ranging in topic from basic concepts and properties, such as floating clouds [21], integrated clouds [23], two-dimensional clouds [55], virtual clouds [7] and expectation curve [29], to backward cloud generator algorithm [44] and applications [8,17,23,24,28,33,34,37,57,58]. Some popular methods for the application of cloud models to linguistic MCDM have recently been proposed [47,49]. The key to linguistic decision-making based on cloud models is the transformation between linguistic variables and clouds, for which Wang proposed a method of generating five clouds on the basis of the golden ratio, but this method has a severe limitation: It can only process linguistic term set of 5 labels [49].

This paper defines a linguistic assessment scale that can be used to perform the transformation between the clouds and linguistic variables of any linguistic term set of odd labels rather than only sets of five labels. This scale can be used to transform linguistic assessment information into numerical assessment values. Then the linguistic MCDM problems can be addressed using the cloud model. In the process of linguistic decision-making, there should be an aggregating step for obtaining the overall evaluation information, and the aggregation operator is one of the most popular tools for this step. A great deal of work has already been done, including (1) linguistic symbolic aggregation operators based on the indices of linguistic labels [40,41,52,54]; (2) linguistic aggregation operators that make computations with fuzzy numbers [3,12]; (3) linguistic aggregation operators based on 2-tuple linguistic model [14,36,43,50]; (4) development by Zhou et al. of the type-1 OWA operator as an extension of Yager's OWA operator for use in the aggregation of uncertain information with uncertain weights [59]. Zhou et al. also developed $\alpha$-level type-1 OWA operators to improve computing efficiency [60,61] and identified the linguistic weights associated with the type-1 OWA operator by proposing the type-2 linguistic quantifiers [62]. They even introduced type-2 OWA operators to tackle the uncertain information which is modeled by type-2 fuzzy sets [63]. The cloud model, which is the representation of qualitative concepts, can be regarded as another new method of addressing linguistic decision-making problems. The aggregation operators of the cloud model play a significant role in the decision-making process. For this reason, some cloud aggregation operators will be introduced in the following text.

The rest of this paper is organized as follows. Section 2 provides the definitions, operations, and methods of comparison of the clouds. Section 3 presents the method of conversion between linguistic variables and clouds, which transforms the linguistic term set into a series of adjacent clouds. Cloud aggregation operators that can be used to aggregate cloud information are presented in Section 4. In Section 5, a multi-criteria group decision-making (MCGDM) method based on cloud aggregation operators is proposed. Section 6 presents an example of a MCGDM problem to illustrate the applicability and effectiveness of the proposed method. The results of the illustrative example shown using the proposed method are compared to those produced using the other three methods. Some summary remarks are given in Section 7.

## 2. Preliminaries

Applying the cloud model to linguistic decision-making involves the operations and properties of clouds. In this section, the definitions and some of the operations and properties of the clouds are outlined.

**Definition 1** [20]. Let $U$ be the universe of discourse and $T$ be a qualitative concept in $U$. $X \subseteq U$. If $x$ ($x \in X$) is a random instantiation of concept $T$ that satisfies $x \sim N(Ex, En'^2)$ and $En' \sim N(En, He^2)$ and $y \in [0,1]$ is the certainty degree of $x$ belonging to $T$, as follows,