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Bimigrativity of binary aggregation functions

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ABSTRACT

We introduce the notions of bimigrativity and total bimigrativity of an aggregation function w.r.t. another aggregation function, as a natural generalization of the notions of migrativity and bisymmetry. We investigate the role of the presence of neutral or absorbing elements. We also pay attention to the class of weighted quasi-arithmetic means, a well-known class of bisymmetric aggregation functions.

1. Introduction

In recent years, there has been a growing interest in the study of the notion of α-migrativity and generalizations thereof [3,8,9,11–13,15,17,16,24,20]. First introduced by Durante and Sarkoci [9] (see also [11]), the α-migrativity property expresses that the result of modifying one of the inputs of a binary aggregation function, by multiplying it by a constant $\alpha \in [0,1]$, is the same regardless of which of the inputs is modified. In the special case when this property holds for any $\alpha \in [0,1]$, we talk about migrativity. This notion was exhaustively analyzed in [7], where it was shown that migrative aggregation functions can be essentially characterized as perturbations of the usual algebraic product $T_{\mathbf{P}}$ by means of a suitable mapping $g : [0,1] \to [0,1]$. From an application point of view, migrativity turns out to be an interesting property in decision making [18,19,21] and in image processing [6]. Closely related topics are covered in, for example, the study of functional equations [4,10].

Furthermore, the modification of the inputs may also be realized by an aggregation function other than the algebraic product. A binary aggregation function A is then called α-B-migrative, with $\alpha \in [0,1]$, w.r.t. an aggregation function B if the identity $A(B(x,\alpha),y) = A(x,B(\alpha,y))$ holds for any $x, y \in [0,1]$. In the special case when this property holds for any α in $[0,1]$, we talk about B-migrativity. This notion was introduced in [5] and subsumes both associativity and migrativity.

The main goal of this paper is to go one step beyond the above. Instead of modifying just one of the inputs by means of an aggregation function and a constant α, we now consider the situation where both inputs are simultaneously modified by means of the same aggregation function, yet through the use of two (possibly different) constants a and b. Explicitly, a binary

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aggregation function A is called (a,b)-B-bimigrative, with a, $b \in [0,1]$, w.r.t. an aggregation function B if the identity $A(B(x,a),B(b,y)) = A(B(x,b),B(a,y))$ holds for any $x,y \in [0,1]$. This naturally leads to the new notions of local B-bimigrativity (when the above identity holds for some specific $(a,b) \in [0,1]^2$) and global B-bimigrativity (when the above identity holds for any $(a,b) \in [0,1]^2$). This approach constitutes a general framework in which bisymmetry [1], B-migrativity and associativity are contained as special cases.

This paper is organized as follows. After some preliminary definitions in Section 2, Section 3 is devoted to the study of bimigrativity from a local point of view, while Section 4 analyses global bimigrativity. In Section 5, we deal with the class of weighted quasi-arithmetic means. We conclude the paper with an even more general notion called total bimigrativity.

2. Preliminaries

We start by recalling some basic notions on aggregation functions that will be needed further on (for more details, see [14]).

Definition 1. An (n-ary) aggregation function is an increasing mapping $A : [0,1]^n \to [0,1]$ such that $A(0,\ldots,0) = 0$ and $A(1,\ldots,1) = 1$.

Unless indicated otherwise, all aggregation functions considered in this paper are binary. The diagonal section of an aggregation function A is the increasing mapping $\delta_A : [0,1] \to [0,1]$ defined by $\delta_A(x) = A(x,x)$. Note that if A is continuous, then δ_A is surjective.

Example 1.

(i) The greatest aggregation function A^* is defined by

$$A^*(x,y) = \begin{cases} 0, & \text{if } x = y = 0, \\ 1, & \text{otherwise.} \end{cases}$$

(ii) The smallest aggregation function A_* is defined by

$$A_*(x,y) = \begin{cases} 1, & \text{if } x = y = 1, \\ 0, & \text{otherwise.} \end{cases}$$

Consider an increasing mapping $g : [0,1] \to [0,1]$ such that $g(0) = 0$ and $g(1) = 1$ (i.e., a unary aggregation function). We can transform any aggregation function A into an aggregation function A_g defined by

$$A_g(x,y) = A(g(x),g(y)).$$

If the mapping g is bijective, it is called an automorphism.

The dual of an aggregation function A is the aggregation function A^d defined by

$$A^d(x,y) = 1 - A(1-x, 1-y).$$

Definition 2. Consider an aggregation function A.

(i) An element $e \in [0,1]$ is called a neutral element of A if $A(x,e) = A(e,x) = x$ for any $x \in [0,1]$.
(ii) An element $a \in [0,1]$ is called an absorbing element (or annihilator) of A if $A(x,a) = A(a,x) = a$ for any $x \in [0,1]$.

Definition 3 [2]. An aggregation function A is called:

(i) associative if the identity

$$A(A(x,y),z) = A(x,A(y,z)),$$

holds for any $x,y,z \in [0,1]$;

(ii) bisymmetric if the identity

$$A(A(x,u),A(y,v)) = A(A(x,y),A(u,v)),$$

holds for any $x,y,u,v \in [0,1]$.

The starting point for the discussion in this paper is the notion of migrativity and generalizations thereof.