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Markov-binary visibility graph: A new method for analyzing complex systems

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In order to study the dynamics of complex systems has been used information recorded in the form of time series. These time series can be investigated from a complex network perspective. Using two-state Markov chain and the binary visibility graph, we investigate these time series. Moreover, several topological aspects of the constructed graph, such as degree distribution, clustering coefficient, and mean visibility length are studied. Our results show that the Markov-binary visibility algorithm stands as a simple method to discriminate statistically dependent and independent systems. Some remarkable examples confirm the reliability of Markov-binary visibility graph for time series analysis.

1. Introduction

Describing the dynamics of complex systems using time series is a fundamental problem in a wide variety of fields that there are different measures to analyze the complex, such as Lyapunov exponent, entropies, and correlation dimension [33]. Each of the above can only deal with a view to analyze the dynamics but complex networks can deal with different aspects to analyze the dynamics [15,33]. In the past few years, numerous transformations to analysis of the dynamics of complex systems based on complex networks have been proposed [15,33,20,17,7,9,14,24,25,31]. Complex network theory has fascinated much attention in the study of social, informational, technological and biological systems, resulting in a deeper understanding of complex systems [33,16,2,5,10,11,21]. One of these transformations is binary visibility algorithm. The binary visibility algorithm is applied in various fields such as stock market indices [22,29,34], human heartbeat dynamics [8,23], energy dissipation rates in three-dimensional fully developed turbulence [19], and foreign exchange rates [32]. The binary visibility algorithm transforms a time series into a binary visibility graph [1]. We are used geometrical visibility of top of the bars in the binary sequence bar diagram to produce binary visibility graphs of time series. On the other hand, there is a linear interpolation between data which have mutual visibility. This algorithm depends on the statistical persistence of the time series [30]. Therefore, this dependence is less, if visibility be low. In this way, we introduce the application of complex networks in testing the randomness of binary sequences. Another method uses of fluctuations of the time series's line diagram for time series analysis. On the other words, we transform fluctuations of the time series's line diagram into two-state Markov chain and then using binary visibility graph, we are mapped the two-state Markov chain into complex networks. This algorithm is defined as a Markov-binary visibility algorithm (MBVA). Whereas this algorithm uses the two-state Markov chains for transform the time series into the complex networks and in a two-state Markov chain, the next state only depends on the current state and not on the sequence of events that preceded it (memoryless), thus, this algorithm is less dependent on the statistical persistence of the time series. Consequently, this algorithm be obtained more precise results. The rest of

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this paper is organized as follows: In Section 2 we introduce the MBVG algorithm. In Section 3 we derive results for statistical properties of the MBVG such as degree distribution, clustering coefficient, visibility length and mean visibility. In Section 4, numerical results of the model are reported with an analysis of the human heartbeat dynamics. The conclusions are given in Section 5.

2. Structure of Markov-binary visibility graph

A Markov chain is a discrete-time process for which the future behavior, given the past and the present, only depends on the present and not on the past. Also, a Markov chain is characterized by a set of states S and the transition probabilities $P_{i\to j}$ between the states. Using this property, we show how Markov-binary visibility algorithm (MBVA) maps the time series into a complex network (see Fig. 1). Here, we briefly describe the Markov-binary visibility algorithm:

First, consider $\{y_i\}_{i=1,\dots,N}$ be a time series of N data. Next, to draw line diagram corresponding to time series. Then, we get slope (m) between any two consecutive data in the time series's line diagram (see Fig. 1). Now, the set of states S of Markov chain is defined as follows:

For the stochastic systems,

$$S = \begin{cases} 1 & m > 0 \\ 0 & m \leqslant 0 \end{cases} \tag{1}$$

and for the deterministic systems,

$$S = \begin{cases} 0 & m \geqslant 0 \\ 1 & m < 0 \end{cases} \tag{2}$$

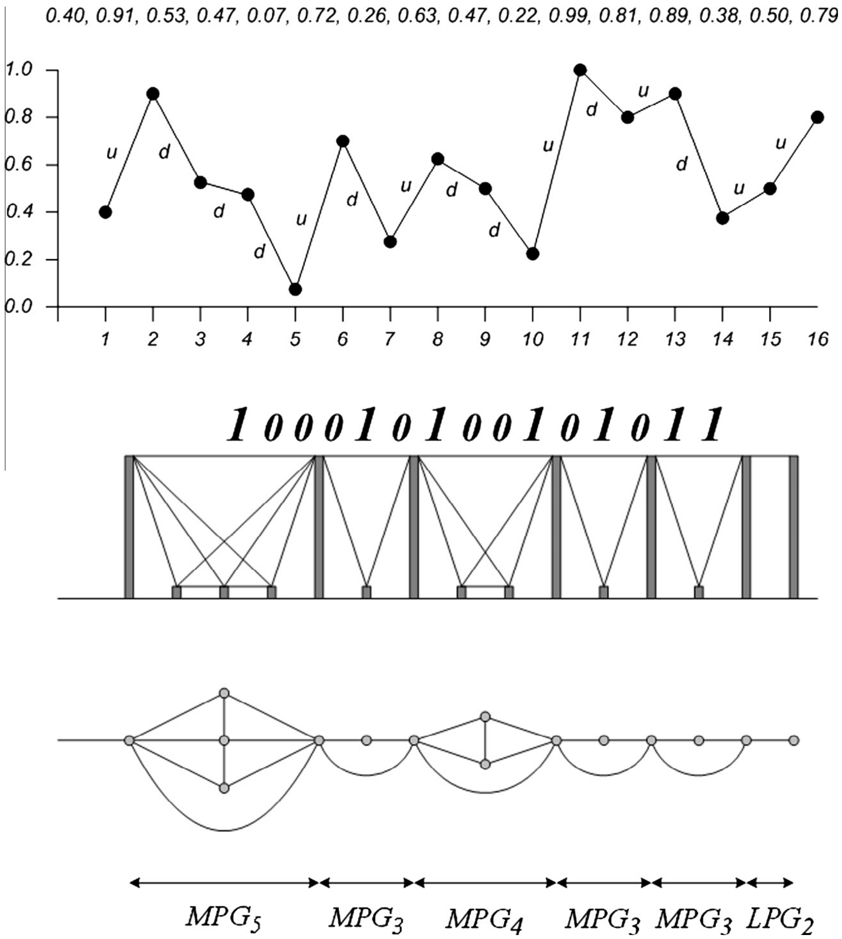

Fig. 1. An example of the Markov binary sequence depicted in the upper part. In the lower part we represent the graph generated through the binary visibility algorithm. The algorithm assigns each bit of the binary sequence to a node in the Markov-binary visibility graph (MBVG). Two nodes i and j in the MBVG are connected if one can draw a visibility line in the binary sequence joining the neighboring x_i and x_j that does not intersect any intermediate bits height.