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Stability analysis of particle dynamics in gravitational search optimization algorithm

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ABSTRACT

In this paper, particle dynamics and stability analysis of gravitational search algorithm (GSA) are investigated. The GSA is a swarm optimization algorithm which is inspired by the Newtonian laws of gravity and motion. Previously, the convergence analysis of the GSA and improved GSA algorithms were presented to demonstrate each particle converges. In this study, the stability of the particle dynamics using Lyapunov stability theorem and the system dynamics concept is analyzed. Sufficient conditions of stability analysis are investigated and utilized for adapting parameters of the GSA. The modified algorithm based on stability analysis is compared with the standard GSA, PSO, RGA, and two methods of improved GSA in terms of average, median, and standard deviation of best-so-far solutions. Simulation results demonstrate the validity and feasibility of the proposed modified GSA. In solving the optimization problem of various nonlinear functions, the high performance is achieved.

1. Introduction

Gravitational search algorithm (GSA) is the newest swarm intelligence technique established by Rashadi et al. [23], inspired by the Newtonian laws of gravity and motion. GSA is a population-based search optimization process which is based on the gravitational interaction between masses. GSA optimizer iteratively simulates the interactions between objects (referred to as particles) which are candidate solutions to the optimization problem at hand based on the laws of gravity and motion. Movements of each object are through a multi-dimensional search space in the influence of gravitation.

GSA is utilized to solve many optimization problems in engineering such as neural network training [19,29], image processing [22, 33], civil and energy engineering [7,14,37], control engineering [3,4,6,21], mechanical engineering [17,18], telecommunication engineering [5,20,25], electronics engineering [28], etc. Moreover, there are review papers published to classify the GSA applications [24], specifically for data clustering and classification [16].

Moreover, a formal convergence of the standard GSA is analyzed in [8].This analysis is based on the behavior of GSA considering mass interactions. In another study [12], a chaotic operator was used to enhance its global convergence to escape from local optima that was named as improved GSA. For converging to the global optima, based on discrete-time linear system theory, convergence analysis of the improved GSA is presented of which probability is one [12].

In this paper, stability analysis of GSA is investigated. First, particle dynamics are represented as a nonlinear feedback controlled system which this representation is similar to the formulated one in [13]. Second, the stability of the particle

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dynamic system is analyzed based on the Lyapunov stability theory that involves with randomness and time varying parameters. In addition, the stability conditions are derived by treating the particle dynamics in the sense of the Lyapunov stability. Although, the convergence proof is a challenging topic, the proof shows that each object converges to a stable point which is the best solution of the particle. The proposed proof on stability also guarantees the convergence of particles. Then, the achieved conditions are used for modifying the standard GSA. The proposed modified algorithm is tested on 20 benchmark functions; unimodal test functions, multimodal test functions, and multimodal test functions with fixed dimensions. The performance of the modified GSA is evaluated. Moreover, stability conditions have been studied for two categories of examples: following and violating stability conditions.

The rest of this paper is organized as follows. In Section 2, the standard GSA is introduced. In Section 3, the dynamic system is formulated and its characteristics are explained. In addition, stability analysis is investigated in this section. Moreover, sufficient stability conditions are explained. In Section 4, the modification of GSA based on stability analysis is developed. In Section 5, results are presented to show the effectiveness of the proposed methods. Results of testing on benchmark functions are presented and compared with the standard GSA. In addition, stability conditions of particles are evaluated. Achievements are discussed in Section 6. At the end, the paper is concluded in Section 7.

2. A brief review of gravitational search algorithm

In this section, a brief review of the standard GSA is presented which is completely explained in [23].

GSA is a population-based heuristic algorithm which is based on Newton's theory of gravity and mass interactions [10,27] to solve optimization problems specifically nonlinear ones. The GSA contains particles (or objects) that interact together through the gravity force. The performance of each object is measured by its masses. All objects move towards other heavier mass objects under the gravity force law. This force triggers a global movement of all particles. Since each object represents a solution of the problem, the heavy mass particles correspond to the best solutions. Algorithm is performed by appropriate adjusting the gravitational and inertia masses. Particles are attracted by the heaviest mass particle during iterations. So, the heaviest particle represents an optimum solution in the search space.

The GSA algorithm is as follows.

Particles position is presented by:

$$X_i = \left(x_i^1, \ldots, x_i^d, \ldots, x_i^n\right) \quad \text{for} \quad i = 1, 2, \ldots, N \tag{1}$$

where x_i^d is the position of the particle i in dimension d. n is the search space dimension and N is the number of particle (the population size). The mass of each particle is calculated as below:

$$M_i = \frac{m_i(t)}{\sum_{j=1}^{N} m_j(t)} \tag{2}$$

where

$$m_i = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)} \tag{3}$$

where fit_i is the fitness value of the particle i at time t. $best(t)$ and $worst(t)$ represent the best and worst fitness of all particles which are respectively defined as follows.

$$best(t) = \min_{j=\{1,\cdots,N\}} fit_j(t) \tag{4}$$

$$worst(t) = \max_{j=\{1,\cdots,N\}} fit_j(t) \tag{5}$$

The force from a set of heavier mass particles applied on a particle is calculated based on the law of gravity as follows.

$$F_i^d(t) = \sum_{j\in kbest, j\neq i} rand_j G(t) \frac{M_j(t) \times M_i(t)}{R_{ij}(t)^{rPower} + \varepsilon} \left(x_j^d(t) - x_i^d(t)\right) \tag{6}$$

where $G(t)$ is the gravitational constant at time t.$rand_j$ is a random number in (0, 1]. M_i and M_j are the mass of particles i and j. Also, R_{ij} is the Euclidean distance between two particles i and j. $rPower$ is the power of distance. $rPower$ is a challenging parameter which is one in [23] and two in the Newtonian equation. Similar to [23], in this study the value of $rPower$ is considered to be 1.

ε is a small value. $kbest$ is a set of the first k particles with the best fitness value and the biggest mass, which is a function of time in such that it is initialized to k_0 at the beginning and decreased with time. So, k_0 is set to N (the number of particles) and is decreased linearly to 1.

The acceleration of the particles i at time t in direction, $a_i^d(t)$ is calculated by:

$$a_i^d(t) = \frac{F_i^d(t)}{M_i(t)} = \sum_{j\in kbest, j\neq i} rand_j G(t) \frac{M_j(t)}{\|X_i(t), X_j(t)\|_2 + \varepsilon} \left(x_j^d(t) - x_i^d(t)\right) \tag{7}$$