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Novel stability conditions for discrete-time T–S fuzzy systems: A Kronecker-product approach

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ABSTRACT

This paper is concerned with the issue of developing a novel strategy to reduce the conservatism of stability conditions for discrete-time Takagi–Sugeno (T–S) fuzzy systems. Unlike the previous ones which are almost quadratic with respect to the state vector, a new class of Lyapunov functions is proposed which is quadratic with respect to the Kronecker products of the state vector, thus including almost the existing ones found in the literature as special cases. By combining the characterizations of homogeneous matrix polynomials and the properties of membership functions, relaxed stability conditions are derived in the form of linear matrix inequalities which can be efficiently solved by the convex optimization techniques. Finally, a numerical example is provided to illustrate the effectiveness of the proposed approach.

1. Introduction

The Takagi–Sugeno (T–S) fuzzy systems [28] have been attracting growing attention in the past years [2–15,17–22,24–41] since they can approximate a wide class of nonlinear dynamic systems in a compact set. In the early literature, the so-called quadratic Lyapunov function method [29] was a popular way to derive stability conditions. This method always makes the stability results much conservative because it requires a common positive definite matrix to satisfy a number of different conditions. Therefore, it is significant to find advanced Lyapunov function methods to obtain less conservative stability conditions.

In [6,11], the piecewise Lyapunov functions were proposed to tackle the stability analysis problem for T–S fuzzy systems, provided that the state space could be partitioned into a number of subspaces. In [3,7], another class of more versatile and powerful Lyapunov functions, called non-quadratic Lyapunov functions, was developed, which includes the quadratic Lyapunov function as a special case [10,13–15,19,26,32]. More recently, [19] further studied the non-quadratic Lypapunv functions and gave a general framework for their application. Additionally, regarding the discrete-time special case, a novel approach called the k-sample variation approach was proposed in [12], whose main idea is to replace the classical 1-sample variation of the Lyapunov function by the variation over several samples. Obviously, the idea is of universal significance.

As the simplest non-quadratic Lyapunov function, the fuzzy Lyapunov function was proposed in [7,30] where the Lyapunov matrix is the one-dimensional fuzzy summation of a set of symmetric matrices weighted by the membership functions (MFs). The authors in [3] further extended the results in [7,30] by setting the Lyapunov matrix as the two-dimensional

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fuzzy summation. In recent years, the Lyapunov matrix has been generally constructed as the multi-dimensional fuzzy summation [4,13,19,40]. Almost at the same time, a more compact form of Lyapunov functions called homogeneous polynomially parameter-dependent (HPPD) Lyapunov functions were proposed in [5,20,34,38] where the Lyapunov matrix is the homogeneous fuzzy summation of a set of symmetric matrices. It is worth noting that the multi-dimensional fuzzy summation and the homogeneous fuzzy summation are in essence the same with respect to the relaxation of the stability conditions but the latter is in a more compact form with a smaller number of decision variables [33].

It should be pointed out that the aforementioned Lyapunov functions are quadratic with respect to the state vector. Even though asymptotically necessary and sufficient (ANS) conditions are obtained by combining Polya's theorem, they could be further relaxed since they are derived based on special Lyapunov functions. In other words, these conditions are ANS only in the sense of special Lyapunov functions which are quadratic with respect to the state vector. It will be seen in the numerical example in the sequel that the degrees of the state variables in Lyapunov functions play a very important role in reducing the conservatism of stability conditions.

Motivated by the aforementioned discussions, this paper further investigates the stability of discrete-time T–S fuzzy systems by employing a new class of Lyapunov functions, which is quadratic with respect to the Kronecker products of the state vector. The proposed Lyapunov functions significantly enlarge the freedom of stability analysis for discrete-time T–S fuzzy systems by considering more information of the state vector. The corresponding ANS conditions are derived as well by applying Polya's theorem and the properties of MFs. The remainder of this paper is arranged as follows. Section 2 introduces preliminaries and backgrounds; the main results are presented in Section 3; a numerical example is given in Section 4; a conclusion is provided in Section 5.

Notations. Throughout this paper, a star $(*)$ denotes the transposed term in a symmetric matrix. $\mathcal{I}_r$ denotes the set $\{1, 2, \ldots, r\}$. $X_{z(t)}$ and $X_{z^+(t)}$ denote $\sum_{i=1}^{r} h_i(z(t))X_i$ and $\sum_{i=1}^{r} h_i(z(t+1))X_i$, respectively. The notation $X \geq Y$ (respectively, $X > Y$) means that X–Y is positive semi-definite (respectively, positive definite) when X and Y are real symmetric matrices or that every element of X–Y is nonnegative (respectively, positive) when X and Y are real vectors. $A \otimes B$ and $A^{\otimes m}$ denote Kronecker product of matrices A and B and mth Kronecker power $A \otimes A \otimes \cdots \otimes A$ (m times), respectively. $\mathbb{N}$ and $\mathbb{Z}$ denote the sets of the nonnegative and positive integers, respectively. $d!$ means factorial operation, i.e., $d! = d \times (d-1) \times \cdots \times 1$.

2. Preliminaries and backgrounds

2.1. Homogeneous matrix polynomials

The following definitions and notations associated with homogeneous matrix polynomials are consistent with those in [5,20,34,38].

Let $P_g(h)$ be a homogeneous matrix polynomial of degree g in $h \in \mathbb{R}^r$, which is defined as

$$P_g(h) \triangleq \sum_{k \in \mathcal{K}_r(g)} h^k P_k \tag{1}$$

where $h^k = h_1^{k_1} h_2^{k_2}, \ldots, h_r^{k_r}$, $h \in \triangle_r$, $k = k_1 k_2, \ldots, k_r$, are the monomials; $P_k \in \mathbb{R}^{n \times n}$, $k \in \mathcal{K}_r(g)$, are matrix-valued coefficients; $\triangle_r \triangleq \{h \in \mathbb{R}^r | \sum_{i=1}^{r} h_i = 1, h \geq 0\}$; $\mathcal{K}_r(g)$ is the set of r-tuples obtained as all possible combinations of nonnegative integers k_i, $i \in \mathcal{I}_r$, such that $k_1 + k_2 + \cdots + k_r = g$. Here, $J_r(g)$ is defined as the number of the elements in $\mathcal{K}_r(g)$, i.e., $J_r(g) = \frac{(r+g-1)!}{(r-1)!g!}$. The usual operations of summation $k + k'$ and subtraction $k - k'$ are defined componentwise. And one writes $k \geq k'$ if $k_i \geq k'_i$, $\forall i \in \mathcal{I}_r$. A special r-tuple $e_i \in \mathcal{K}_r(1)$ is written as $e_i = 0 \ \cdots \ 0 \underbrace{1}_{i\text{th}} 0 \ \cdots \ 0$. Especially, $P_0(h) \triangleq P$, $P_1(h) \triangleq P_{z(t)}$ (let $P_{e_i} = P_i$).

For simplicity of notations, the following shortenings will be used in the sequel:

$$h_i(t) = h_i(z(t)), \quad h_i = h_i(t), \quad A_z = A_{z(t)}, \quad h = [h_1, \ldots, h_r]^T, \quad x = x(t),$$
$$\pi(k) = k_1! k_2!, \ldots, k_r!, \quad h_i^+ = h_i(t+1), \quad h^+ = [h_1^+, \ldots, h_r^+]^T, \quad \mathcal{K}(g) = \mathcal{K}_r(g).$$

With the aforementioned definitions, the following lemma is obtained by summarizing some results in [5,20,34,38], which is very useful in the development of the main results.

Lemma 1. *Let $F_g(h)$ defined in (1), $g, d, d_1, d_2 \in \mathbb{N}$. Then, the following equations hold:*

$$\left(\sum_{i=1}^{r} h_i \right)^d = \sum_{k \in \mathcal{K}(d)} h^k \frac{d!}{\pi(k)}, \tag{2}$$

$$\begin{aligned} \left(\sum_{i=1}^{r} h_i \right)^d F_g(h) &= \sum_{k \in \mathcal{K}(g+d)} h^k \sum_{k' \in \mathcal{K}(g), k \geq k'} \frac{d!}{\pi(k-k')} F_{k'} \\ &= \sum_{k \in \mathcal{K}(g+d)} h^k \sum_{k' \in \mathcal{K}(d), k \geq k'} \frac{d!}{\pi(k')} F_{k-k'}, \end{aligned} \tag{3}$$