Information Sciences 264 (2014) 196–210

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Pessimistic rough set based decisions: A multigranulation fusion strategy ☆

Yuhua Qian [a], Shunyong Li [b], Jiye Liang [a,*], Zhongzhi Shi [c], Feng Wang [a]

[a] Key Laboratory of Computational Intelligence and Chinese Information Processing of Ministry of Education, Shanxi University, Taiyuan 030006, Shanxi, China
[b] School of Computer and Information Technology, Shanxi University, Taiyuan 030006, Shanxi, China
[c] Institute of Computing Technology Chinese Academy of Sciences, 100190 Beijing, China

ARTICLE INFO

Article history:
Received 15 January 2013
Received in revised form 25 September 2013
Accepted 20 December 2013
Available online 31 December 2013

Keywords:
Rough set
Granular computing
Multigranulation
Pessimistic rough set based decision
Attribute reduction

ABSTRACT

Multigranulation rough sets (MGRS) is one of desirable directions in rough set theory, in which lower/upper approximations are approximated by granular structures induced by multiple binary relations. It provides a new perspective for decision making analysis based on rough set theory. In decision making analysis, people often adopt the decision strategy "Seeking common ground while eliminating differences" (SCED). This strategy implies that one reserves common decisions while deleting inconsistent decisions. From this point of view, the objective of this study is to develop a new multigranulation rough set based decision model based on SCED strategy, called pessimistic multigranulation rough sets. We study this model from three aspects, which are lower/upper approximation and their properties, decision rules and attribute reduction, in this paper.

1. Introduction

Rough set theory, originated by Pawlak [20,21], has become a well-established theory for uncertainty management in a wide variety of applications related to pattern recognition, image processing, feature selection, neural computing, conflict analysis, decision support, data mining and knowledge discovery [2,4,10,11,15,16,25–28,30,31,34,41,43]. One of the strengths of rough set theory is the fact that all its parameters are obtained from the given data. In other words, instead of using external numbers or other additional parameters, *the rough set data analysis* (RSDA) utilizes solely the granular structure of the given data, expressed as classes of suitable equivalence relations [5,6,25,26,33].

In the past ten years, several extensions of the rough set model have been proposed in terms of various requirements, such as the probabilistic rough set model (see [38]), the variable precision rough set (VPRS) model (see [42,45]), the rough set model based on tolerance relation (see [12–14]), the Bayesian rough set model (see [32]), the Dominance-based rough set model (see [3]), game-theoretic rough set model (see [7,8]), the fuzzy rough set model and the rough fuzzy set model (see [1,19]). In particular, the probabilistic rough sets have been paid close attention [9,35–37,39]. A special issue on probabilistic rough sets was set up in International Journal of Approximate Reasoning, in which six relative papers were published [36]. Yao presented a new decision making method based on the probabilistic rough set, called three-way decision, which are constructed by positive region, boundary region and negative region, respectively [39]. In the literature [37], the author

☆ This is an extended version of the paper presented at the Second International Workshop on Rough Sets Theory, Zhoushan, China.

* Corresponding author. Tel./fax: +86 0351 7018176.
E-mail addresses: jinchengqyh@126.com (Y. Qian), lisy75@sxu.edu.cn (S. Li), ljy@sxu.edu.cn (J. Liang), shizz@ics.ict.ac.cn (Z. Shi), sxuwangfeng@126.com (F. Wang).

0020-0255/$ - see front matter © 2013 Published by Elsevier Inc.
http://dx.doi.org/10.1016/j.ins.2013.12.014

further emphasized the superiority of three-way decisions in probabilistic rough set models. In fact, the probabilistic rough sets is developed based on the Bayesian decision principle, in which its parameters can be learned from a given decision table. Three-way decisions are most of superiorities of probabilistic rough set models. Since the fuzzy rough sets was proposed, this theory have been also largely developed [10,31,44]. Jensen and Shen [10] developed a series of feature selection approaches for classification using the fuzzy rough set model. Shen and Chouchoulas [31] presented a rough-fuzzy approach for generating classification rules. Zhao et al. [44] addressed a fuzzy variable rough set model and trained a rule-based classifier with it. Indeed, the fuzzy rough set theory can do better than other rough set models when we need to deal with numeric data, while the classical rough sets will collapse. In fact, the classical rough set theory will be dominant when all of the attribute values and concepts are discrete-valued.

In the view of granular computing (proposed by Zadeh [40]), in existing rough set models, a general concept described by a set is always characterized via the so-called upper and lower approximations under a single granulation, i.e., the concept is depicted by known knowledge induced from a single relation (such as equivalence relation, tolerance relation and reflexive relation) on the universe [17,18,22,38]. Conveniently, this kind of rough set models is called single granulation rough sets, just SGRS. However, this approach to describing a target concept is mainly based on the following assumption:

If P and Q are two sets of conditional features and $X \subseteq U$ is a target concept, then the rough sets of X are derived from the quotient set $U/(P \cup Q)$. In fact, the quotient set is equivalent to the formula

$$\widehat{P \cup Q} = \{P_i \cap Q_j : \ P_i \in U/P, \ Q_j \in U/Q, \ P_i \cap P_j \neq \emptyset\}.$$

It implies the following two ideas:

(1) we can perform an intersection operation between any P_i and Q_j,
(2) the target concept is approximately described by using the quotient set $U/(P \cup Q)$.

In fact, the target concept is described by using a finer granulation (partitions) formed through combining two known granulations (partitions) induced from two attribute subsets. Although it generates a much finer granulation, the combination/fining destroys the original granular structure/partitions. In general, the above assumption cannot be always satisfied or required in practice. In many circumstances, we often need to describe concurrently a target concept through multi binary relations (e.g. equivalence relation, tolerance relation, reflexive relation and neighborhood relation) on the universe according to a user's requirements or targets of problem solving. Based on this consideration, Qian et al. [23–25] introduced multigranulation rough set theory (MGRS) to more widely apply rough set theory in practical applications, in which lower/upper approximations are approximated by granular structures induced by multiple binary relations.

Besides the motivation of theoretical study above, it also has the motivation of real application. From the viewpoint of rough set's application, the multigranulation rough set theory is very desirable in many real applications, such as multi-source data analysis, knowledge discovery from data with high dimensions and distributive information systems. For example, It is very desirable to develop multigranulation rough sets in the following two cases.

(1) When we apply the rough set theory for data mining and knowledge discovery from multi-source data, its key task is to consider how to knowledge representation and rough approximation in the context of multi-source information systems. In order to efficiently discover knowledge online, it is unnecessary to gather and combine every information systems from multiple sources as an entire information system for data analysis. More reasonable strategy is to directly analyzing these multi-source information systems. In this situation, the classical single granulation rough set theory (SGRS) has its limitation that the computational times of algorithms are too longer to efficiently knowledge discovery from multi-source information systems.
(2) When analyzing data with high dimensions, a lot of attributes bring out a challenge for knowledge discovery. There are two main problems: (1) after granulating data using all attributes, the intensions of information granules obtained will be very longer and the extensions of those will be very smaller, which determines a rule-based classifier with much smaller generalization ability; and (2) a lot of attributes also lead to inefficient of algorithms in rough set theory. These two shortcomings are so important that the existing rough set models cannot be well used to rough set-based data analysis for data with high dimensions.

In the multigranulation rough set theory, each of various binary relation determines a corresponding information granulation, which largely impacts the commonality between each of the granulations and the fusion among all granulations. In this paper, we do not further discuss how binary relations impact information fusion among all granulations,[1] but develop a new decision method, called a pessimistic multigranulation rough set model.

Classical rough sets and multigranulation rough sets are complementary in many practical applications. When two attribute sets in information systems possesses a contradiction or inconsistent relationship, or efficient computation is required,

[1] How each of various binary relations impacts information fusion among information granulations is a very important and interesting issue in the multigranulation rough set theory. This will produce many information fusion methods, especially the information fusion in various kinds of granular spaces. This study is beyond the scope of this paper. We will investigate this issue in further work.